Eq de Hamilton

$$i\hbar \frac{\partial \psi}{\partial t} = H \psi$$

$$A(t) = e^{i/\hbar Ht} A \, e^{-i/\hbar Ht}$$

$$\boxed{\frac{d\hat{A}}{dt} = \frac{1}{i\hbar}[A, H]}$$ for any H even time dependent

$$\frac{d\hat{A}}{dt} = i/\hbar [\hat{H}, \hat{A}]$$

let $H = \frac{\hat{p}^2}{2m} + V(\hat{x})$ $(d=1)$

$$\frac{d\hat{x}}{dt} = \frac{1}{i\hbar}[\hat{x}, \hat{H}] = \frac{1}{i\hbar}[\hat{x}, \frac{\hat{p}^2}{2m}]$$

$$\frac{d\hat{p}}{dt} = \frac{1}{i\hbar}[\hat{p}, V(\hat{x})]$$

$$[\hat{x}, \hat{p}^2] = \hat{p}[x, p] + [x\, p]\, p = i\hbar\, 2\hat{p}$$

$$\boxed{\frac{d\hat{x}}{dt} = \frac{1}{i\hbar}\frac{1}{2m} i\hbar\, 2\hat{p} = \hat{P}/m}$$

$$[\hat{p}, V(\hat{x})] \rightarrow \quad -i\hbar \frac{\partial}{\partial x} V\psi + i\hbar V \frac{\partial \psi}{\partial x} =$$

$$= -i\hbar \frac{\partial V}{\partial x}\psi - i\hbar V \cancel{\frac{\partial \psi}{\partial x}} + i\hbar V \cancel{\frac{\partial \psi}{\partial x}}$$

$$[\hat{p}, V(\hat{x})] = -i\hbar \frac{\partial V}{\partial x} \mathbb{1}$$

$$\boxed{\frac{d\hat{p}}{dt} = -\frac{\partial V(\hat{x})}{\partial \hat{x}}}$$

Hamilton eqs

$$\left.\begin{aligned}\frac{d\hat{x}}{dt} &= \frac{\hat{p}}{m}\\ \frac{d\hat{p}}{dt} &= -\frac{\partial V(\hat{x})}{\partial \hat{x}}\end{aligned}\right\}$$ all in Heisenberg's picture

<u>Example</u>

free particle

$$\begin{cases}\dfrac{d\hat{x}}{dt} = \dfrac{\hat{p}}{m}\\[2mm] \dfrac{d\hat{p}}{dt} = 0\end{cases}$$

integrate

$\hat{p}(t) = \hat{p}(0)$ $\qquad$ $\hat{p}$ is a conserved quantity

integrate

$$\hat{x}(t) = \hat{x}(0) + \frac{1}{m}\int_0^t dt'\, \hat{p}(t')$$

$$\boxed{\hat{x}(t) = \hat{x}(0) + \frac{\hat{p}(0)}{m}\,t}$$ uniform motion

Constant Force

$$V(x) = -F\hat{x}$$

Ehrenfest theorem

$$\begin{cases} \dfrac{d}{dt}\langle \hat{x}(t)\rangle = \dfrac{\langle \hat{p}(t)\rangle}{m} \\[2ex] \dfrac{d}{dt}\langle \hat{p}(t)\rangle = -\left\langle \dfrac{\partial V}{\partial \hat{x}}\right\rangle \end{cases}$$

- example free particle

$$\frac{d\langle x(t)\rangle}{dt} = \frac{\langle p(0)\rangle}{m}$$

$$\frac{d\langle p(t)\rangle}{dt} = 0$$

$$\langle x(t)\rangle = \langle x(0)\rangle + \frac{\langle p(0)\rangle}{m} t$$

classical eq of motion

- Constant force

$$\frac{d\langle p(t)\rangle}{dt} = F \qquad \langle p(t)\rangle = \langle p(0)\rangle + Ft$$

$$\frac{d\langle x(t)\rangle}{dt} = \frac{\langle p(t)\rangle}{m} = \frac{\langle p(0)\rangle}{m} + \frac{F}{m} t$$

$$\langle x(t)\rangle = \langle x(0)\rangle + \frac{\langle \hat{p}(0)\rangle}{m} t + \frac{1}{2}\frac{F}{m} t^2$$

- harmonic oscillators

$$V(x) = \frac{1}{2} m\omega^2 \hat{x}^2$$

$$\frac{\partial V}{\partial x} = m\omega^2 \hat{x}$$

$$\begin{cases} \dfrac{d\langle \hat{p} \rangle}{dt} = -m\omega^2 \langle \hat{x} \rangle \\ \dfrac{d\langle x \rangle}{dt} = \dfrac{\langle p \rangle}{m} \end{cases}$$

classical eq of motion

but for non □ $V(x)$ example

$$V(x) = a\hat{x}^4 \qquad a > 0$$

$$\begin{cases} \dfrac{d\langle p(t) \rangle}{dt} = -4a\langle \hat{x}^3 \rangle \\ \dfrac{d\langle \hat{x} \rangle}{dt} = \dfrac{\langle p \rangle}{m} \end{cases}$$

— non classical eq of motion

$$\langle \hat{x}^3 \rangle \neq \langle \hat{x} \rangle^3$$