$$\boxed{\begin{array}{l}\text{Oscillatore Armonico Classico} \\ \text{soluzione usando le matrici di Pauli}\end{array}}$$

eq di Hamilton

$$\begin{cases} \dot{x} = p/m \\ \dot{p} = -m\omega^2 x \end{cases} \qquad \frac{d}{dt}\begin{pmatrix} x \\ p \end{pmatrix} = \begin{bmatrix} 0 & \frac{1}{m} \\ -m\omega^2 & 0 \end{bmatrix}\begin{pmatrix} x \\ p \end{pmatrix}$$

introduciamo una lunghezza ℓ per adimensionalizzare le equazioni

$$\boxed{\xi = x/\ell \qquad \pi = \frac{p}{m\ell\omega}}$$

$$\dot{\xi}\cancel{\ell} = \frac{1}{\cancel{m}}\cancel{m}\cancel{\ell}\omega\pi \qquad\qquad \frac{d}{dt}\begin{pmatrix} \xi \\ \pi \end{pmatrix} = \begin{pmatrix} 0 & \omega \\ -\omega & 0 \end{pmatrix}\begin{pmatrix} \xi \\ \pi \end{pmatrix}$$

$$\cancel{m}\cancel{\ell}\cancel{\omega}\dot{\pi} = -\cancel{m}\omega^{\cancel{2}}\cancel{\ell}\xi$$

$$\boxed{\frac{d}{dt}\begin{pmatrix} \xi \\ \pi \end{pmatrix} = i\omega\underbrace{\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}}_{\sigma_y}\begin{pmatrix} \xi \\ \pi \end{pmatrix}}$$

$$\begin{pmatrix} \xi(t) \\ \pi(t) \end{pmatrix} = e^{i\omega\hat{\sigma}_y t}\begin{pmatrix} \xi(0) \\ \pi(0) \end{pmatrix} =$$

$$= \left(\cos\omega t\, \mathbb{1} + i\,\hat{\sigma}_y \sin\omega t\right)\begin{pmatrix} \xi(0) \\ \pi(0) \end{pmatrix}$$

$$\xi(t) = \xi(0)\cos\omega t + \pi(0)\sin\omega t$$

$$\pi(t) = \pi(0)\cos\omega t - \xi(0)\sin\omega t$$

$$\boxed{\begin{aligned} x(t) &= x(0)\cos\omega t + \frac{p(0)}{m\omega}\sin\omega t \\ p(t) &= p(0)\cos\omega t - m\omega x(0)\sin\omega t \end{aligned}}$$

solutions for harmonic oscillator

le quadrature non sono altro che

gli auto stati di σ_y

$$\boxed{\begin{aligned} a &= \frac{1}{\sqrt{2}}(\xi + i\pi) \\ a^* &= \frac{1}{\sqrt{2}}(\xi - i\pi) \end{aligned}}$$

che possono esprimersi come

$$R = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}$$

$$\begin{pmatrix} a \\ a^* \end{pmatrix} = \underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}}\begin{pmatrix} \xi \\ \pi \end{pmatrix}$$

moltiplicando le eq del moto per questa matrice

ho

$$\frac{d}{dt}\begin{pmatrix} a \\ a^* \end{pmatrix} = i\omega \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\left[\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}\right]^{-1}\begin{pmatrix} \xi \\ \pi \end{pmatrix}$$

$$\left\{\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}\right\}^{-1} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix} = R^{\dagger}$$

che sono le relazioni per le inverse

$$\boxed{\begin{aligned} \xi &= \frac{1}{\sqrt{2}}(a+a^*) \\ \pi &= \frac{1}{\sqrt{2}\,i}(a-a^*) \end{aligned}}$$

$$\begin{pmatrix} \xi \\ \pi \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix}\begin{pmatrix} a \\ a^* \end{pmatrix}$$

autovett $\lambda = -1$ autovett $\lambda = 1$

e quindi

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix} =$$

$$= \frac{1}{2}\begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}\begin{pmatrix} -1 & 1 \\ i & i \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\frac{d}{dt}\begin{pmatrix} a \\ a^* \end{pmatrix} = i\omega\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a \\ a^* \end{pmatrix}$$

$$\boxed{\begin{cases} a(t) = e^{-i\omega t}a(0) \\ a^*(t) = e^{i\omega t}a^*(0) \end{cases}}$$

$$\boxed{\begin{aligned} x(t) &= \frac{\ell}{\sqrt{2}}(a(t)+a^*(t)) \\ p(t) &= \frac{m\ell\omega}{\sqrt{2}\,i}(a(t)-a^*(t)) \end{aligned}}$$