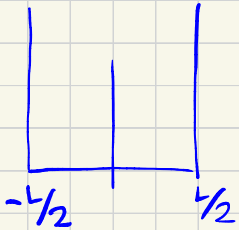
δ nella buca di potenziale



1) $\psi_n(-L/2) = \psi_n(L/2) = 0$

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \lambda\delta(x)\right]\psi_n(x) = E_n\psi_n(x)$$

$$-\frac{\hbar^2}{2m}\left[\psi'(\varepsilon) - \psi'(-\varepsilon)\right] + \lambda\psi_n(0) = E_n\left(\cancel{\psi_n}(0)\right)\varepsilon$$

2) $\psi'(\varepsilon) - \psi'(-\varepsilon) = -\frac{2m\lambda}{\hbar^2}\psi(0)$

i termini dispari non sono modificati

consideriamo le funzioni pari

$\psi = A\cos Kx + B\sin(K|x|)$

$\psi(0) = A$

$\psi' = -AK\sin Kx + BK\cos(K|x|)\,\mathrm{sgn}(x)$

$\Delta\psi'(0) = -2BK$

2) $-\cancel{2}BK = -\frac{\cancel{2}m\lambda}{\hbar^2}A \qquad -\frac{A}{B} = -\frac{\hbar^2 K}{m\lambda}$

1) $A \cos k\frac{L}{2} + B \sin k\frac{L}{2} = 0 \quad x>0$

$$\operatorname{tg} k\frac{L}{2} = -\frac{A}{B} = -\frac{\hbar^2 k}{m\lambda}$$

gli autovalori sono determinati per ogni

$x>0$ (pari)

$$-\frac{\hbar^2}{2m}\psi'' = E\psi$$

$$\psi'' = -Ak^2\cos kx - Bk^2 \sin kx = -k^2\psi$$

$$E = \frac{\hbar^2 k^2}{2m}$$

definiamo le lunghezze di scattering

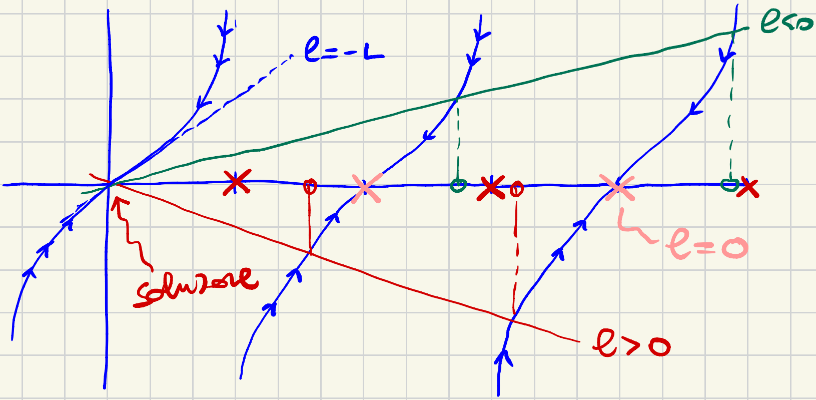
$$\boxed{\ell = \frac{2\hbar^2}{m\lambda}} \qquad [\lambda] = EL$$

$$\operatorname{tg}\left(k\frac{L}{2}\right) = -\frac{k\ell}{2}\,\frac{L}{2}\,\frac{2}{L}$$

$$k\frac{L}{2} = x$$

$$\operatorname{tg}(x) = -x\frac{\ell}{L} \qquad \ell>0 \quad \ell<0$$

dipende da λ



× perturbativo

questo se $\left|\frac{\ell}{L}\right| < 1$

se $\ell/L = \pm 1$