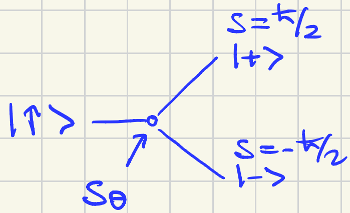
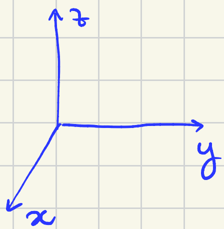
Misure ripetute su di uno spin

Iniziamo con il problema standard



misurando s lungo una direzione θ rispetto a z su di uno stato polarizzato $\uparrow$



$$|+\rangle = e^{-\frac{i}{\hbar}\frac{\hbar}{2}\theta\sigma_y}|\uparrow\rangle$$

$$|-\rangle = e^{-i\frac{\theta}{2}\sigma_y}|\downarrow\rangle$$

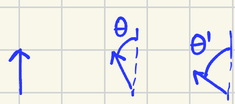
$$|+\rangle = \cos\frac{\theta}{2}|\uparrow\rangle + \sin\frac{\theta}{2}|\downarrow\rangle$$

$$|-\rangle = -\sin\frac{\theta}{2}|\uparrow\rangle + \cos\frac{\theta}{2}|\downarrow\rangle$$

$$P(S_\theta = \tfrac{\hbar}{2}|\uparrow) = |\langle +|\uparrow\rangle|^2 = |\langle\uparrow|+\rangle|^2 = \cos^2\frac{\theta}{2}$$

$$P(S_\theta = -\tfrac{\hbar}{2}|\uparrow) = \sin^2\frac{\theta}{2}$$

Posso adesso fare una altra misura di spin con un altro angolo θ' immediatamente dopo la misura di S_θ

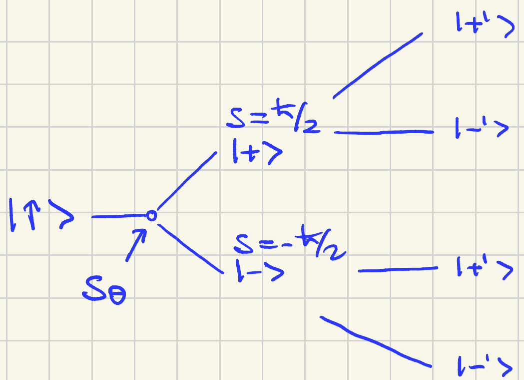


sempre ruotando nel piano xz

definisco $\varphi = \theta' - \theta$ e quindi

$$|+'\rangle = e^{-\frac{i}{2}\theta'\sigma_y}|\uparrow\rangle = e^{-\frac{i}{2}\varphi}|+\rangle = \cos\frac{\varphi}{2}|+\rangle + \sin\frac{\varphi}{2}|-\rangle$$

$$|-'\rangle = e^{-\frac{i}{2}\theta'\sigma_y}|\downarrow\rangle = e^{-\frac{i}{2}\varphi}|-\rangle = -\sin\frac{\varphi}{2}|+\rangle + \cos\frac{\varphi}{2}|-\rangle$$



$$P(+'|+) = |\langle +'|+\rangle|^2 = \cos^2\frac{\varphi}{2}$$

$$P(-'|+) = |\langle -'|+\rangle|^2 = \sin^2\frac{\varphi}{2}$$

$$P(+'|-) = |\langle +'|-\rangle|^2 = \sin^2\frac{\varphi}{2}$$

$$P(-'|-) = |\langle -'|-\rangle|^2 = \cos^2\frac{\varphi}{2}$$

la probabilità di ottenere

$$P(+'|\uparrow) = P(+'|+)P(+|\uparrow) + P(+'|-)P(-|\uparrow)$$

$$= \cos^2\frac{\varphi}{2}\cos^2\frac{\theta}{2} + \sin^2\frac{\varphi}{2}\sin^2\frac{\theta}{2}$$

$$P(-'|\uparrow) = P(-'|+)P(+|\uparrow) + P(-'|-)P(-|\uparrow) =$$

$$= \sin^2\frac{\varphi}{2}\cos^2\frac{\theta}{2} + \cos^2\frac{\varphi}{2}\sin^2\frac{\theta}{2}$$

$$P(+'|\uparrow) + P(-'|\uparrow) = \cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2} = 1$$