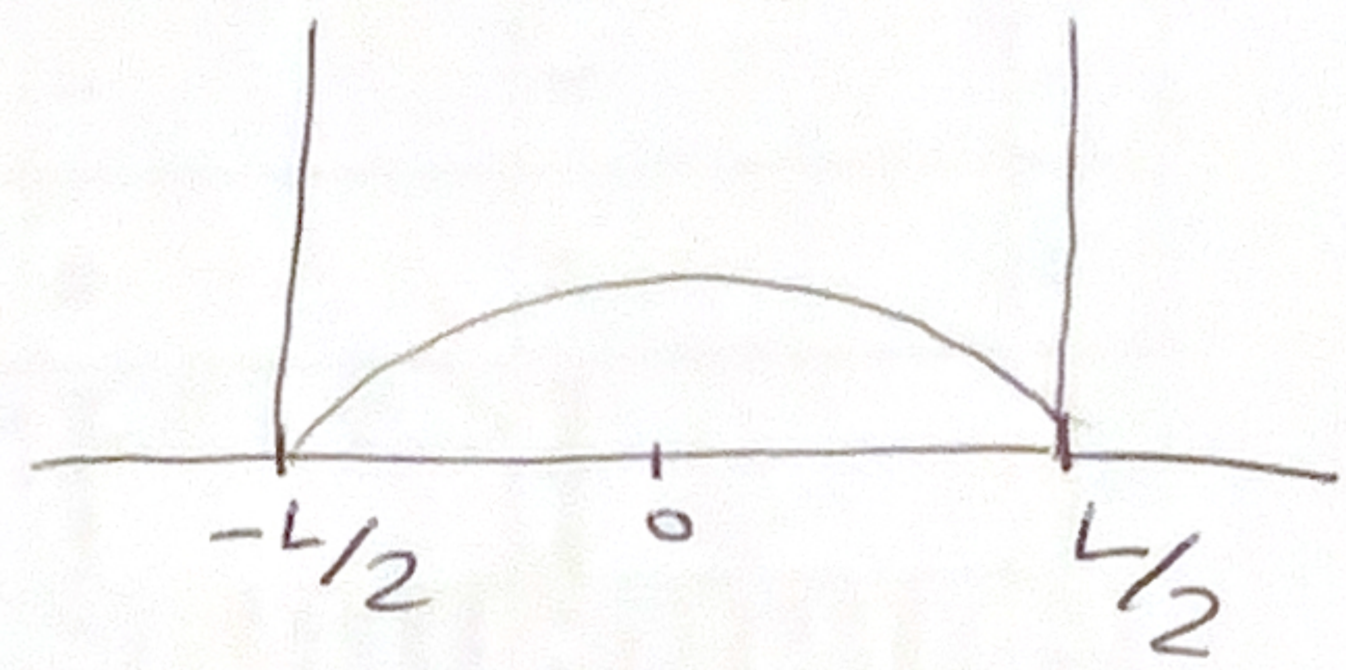
# Wall Removal




$t=0$ $\quad \psi(x,0)=\psi_1(x)$

$$\psi_1(x)=\sqrt{\frac{2}{L}}\cos(k_1 x) \qquad k_1=\frac{\pi}{L}$$

evoluzione temporale

$$\psi_1=\sqrt{\frac{2}{L}}\,\frac{e^{ik_1x}+e^{-ik_1x}}{2}$$

onde piane progressive regressive

dopo la rimozione sicura l'energia

$$E=\frac{\hbar^2}{2m}(k_1)^2=\frac{\hbar^2}{2m}(-k_1)^2$$

$$\psi(x,t)=\sqrt{\frac{2}{L}}\,e^{-\frac{i}{\hbar}Et}\left(\frac{e^{ik_1x}+e^{-ik_1x}}{2}\right)$$

$$|\psi(x,t)|^2=|\psi(x,0)|^2 \qquad ?$$

Soluzione

$$\phi(k) = \langle k|\psi\rangle = \int dx \langle k|x\rangle\langle x|\psi\rangle$$

$$= \int_{-L/2}^{L/2} dx \frac{1}{\sqrt{2\pi}} e^{ikx} \sqrt{\frac{2}{L}} \frac{e^{ik_1 x} + e^{-ik_1 x}}{2}$$

$$= \frac{1}{\sqrt{\pi L}} \int_{-L/2}^{L/2} dx \left[ e^{i(k+k_1)x} + e^{i(k-k_1)x} \right]$$

← non è una $\delta$

$$= \frac{1}{\sqrt{\pi L}} 2 \left\{ \frac{\sin[(k+k_1)L/2]}{(k+k_1)} + \frac{\sin[(k-k_1)\frac{L}{2}]}{k-k_1} \right\}$$

$$k_1 L/2 = \frac{\pi}{L}\frac{L}{2} = \pi/2$$

$$\sin(x+\pi/2) = \cos(x)$$

$$\sin(x-\pi/2) = -\cos(x)$$

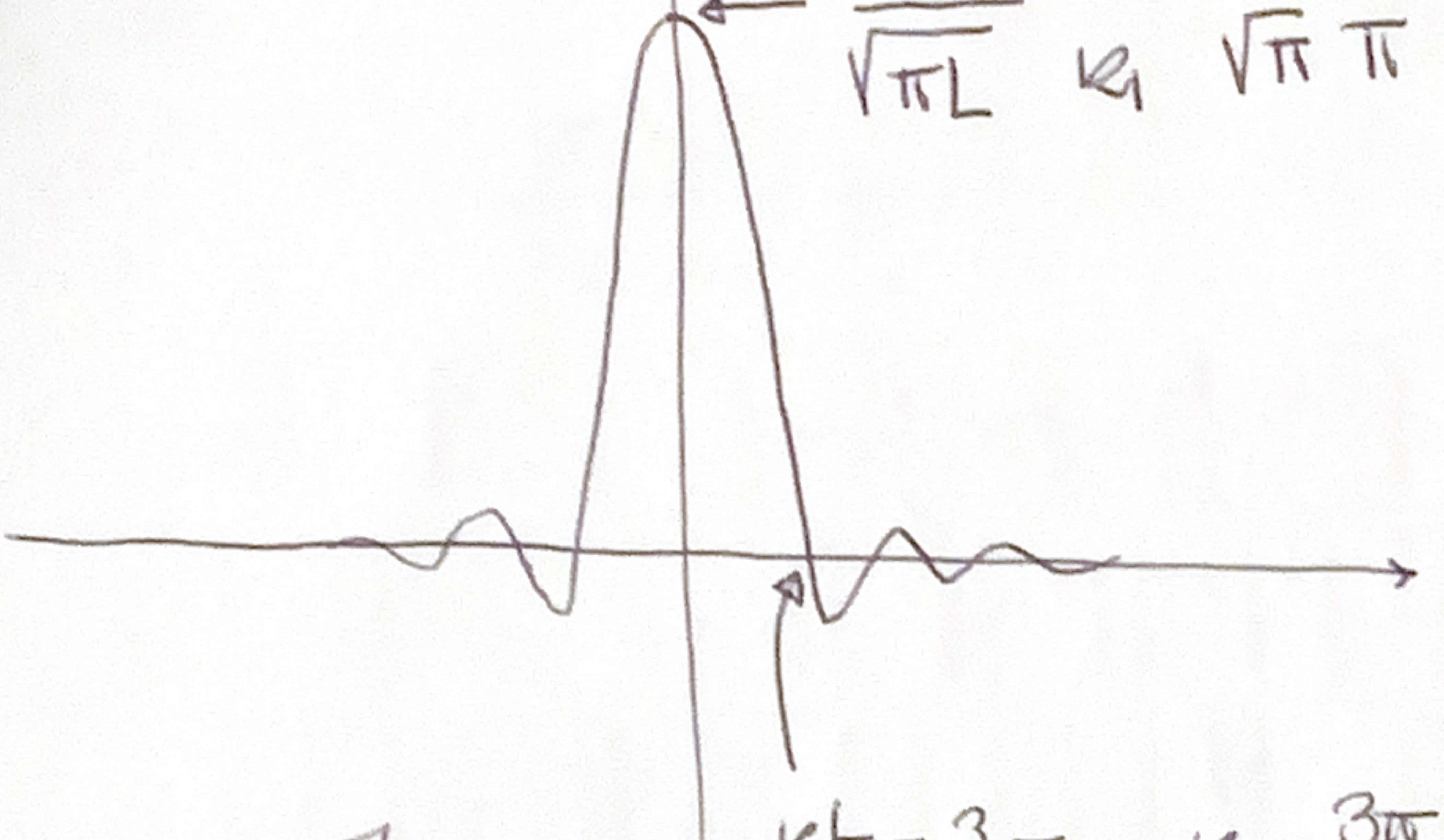
$$\phi(k) = \frac{2}{\sqrt{\pi L}} \left( \frac{1}{k+k_1} - \frac{1}{k-k_1} \right) \cos(k\tfrac{L}{2})$$

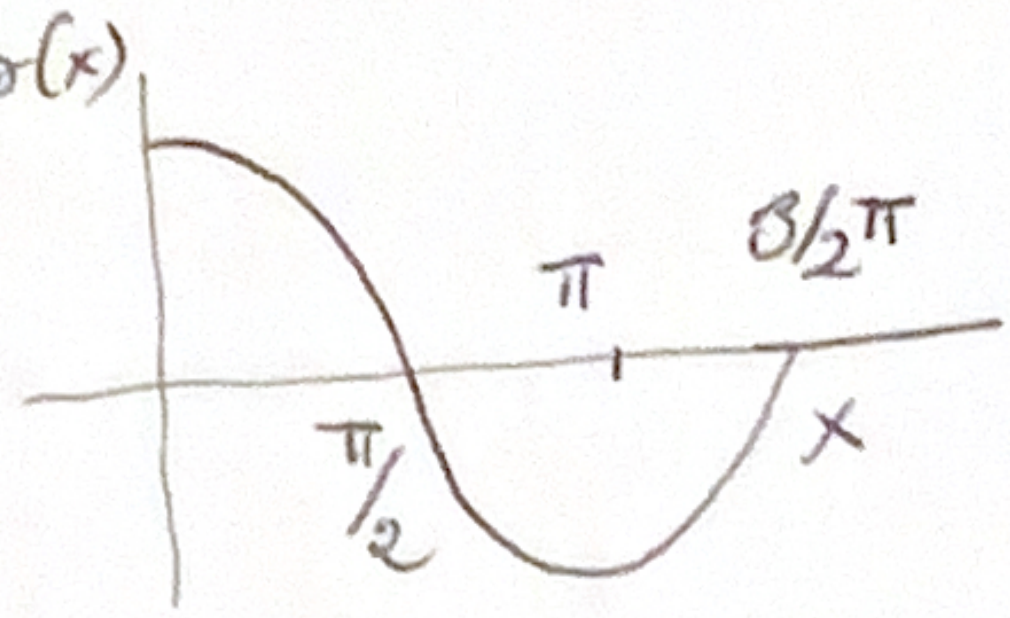
$$k-k_1-k-k_1$$

$$\boxed{\phi(k) = -\frac{4}{\sqrt{\pi L}} \frac{k_1}{k^2-k_1^2} \cos(k\tfrac{L}{2})}$$

$$\cos(\pm k_1 \tfrac{L}{2}) = \cos(\pm \tfrac{\pi}{L}\tfrac{L}{2}) = 0$$

$$\frac{4}{\sqrt{\pi L}} \frac{1}{k_1} = \frac{4}{\sqrt{\pi}} \frac{\sqrt{L}}{\pi}$$

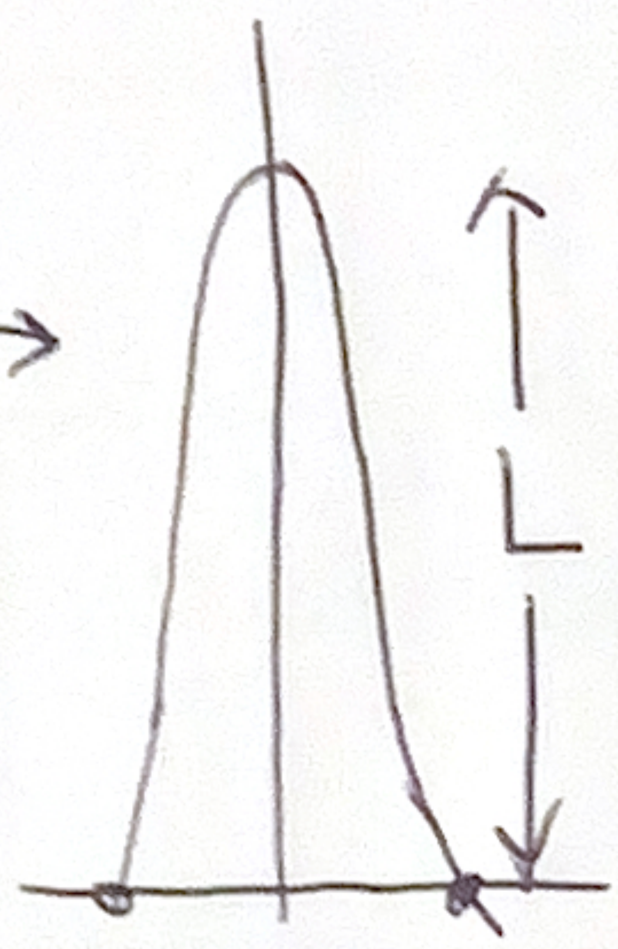





$$k\frac{L}{2} = \frac{3}{2}\pi \qquad k = \frac{3\pi}{L}$$

per $L \to \infty$

$|\phi(k)|^2 \to$




$\sim L$

$$\psi(x,t) = \int dk \, \langle x|k\rangle \langle k|\psi,t\rangle$$

$$= \int_{-\infty}^{+\infty} dk \, \frac{1}{\sqrt{2\pi}} e^{-ikx} \underbrace{\phi(k)\, e^{-\frac{i}{\hbar}\frac{\hbar^2 k^2}{2m}t}}_{\phi(k,t)}$$

$\langle k^2\rangle = \int_{-\infty}^{+\infty} dk \, \phi(k)^2 \, k^2 =$

Calcolo del valor medio di $\hat{p}^2$ sullo stato evoluto nel tempo

$= \frac{16}{\pi L} \int_{-\infty}^{+\infty} dk \, \frac{k_1^2}{(k^2 - k_1^2)^2} \cos^2\left(k\frac{L}{2}\right) k^2$

↑
converge

ma il valore è lo stesso a qualsiasi tempo e coincide con il valore di

$\langle 1|\hat{p}^2|1\rangle$

nello stato fondamentale della buca $\infty$ che è

$\langle 1|\hat{p}^2|1\rangle = 2m E_1 = 2m \frac{\hbar^2}{2m} k_1^2 = \hbar^2 k_1^2 = \hbar^2 \frac{\pi^2}{L^2}$

$$\hat{x}(t) = \hat{x}(0) + \frac{\hat{p}(0)}{m} t$$

$$\hat{x}^2(t) = \hat{x}^2(0) + \frac{\hat{p}(0)^2}{m^2} t^2 + \frac{1}{m}\{x, p\} t$$

$$\langle \hat{x}^2(0) \rangle = \langle 1 | x^2 | 1 \rangle =$$

$$= \frac{2}{L} \int_{-L/2}^{L/2} dx\, x^2 \cos^2(k_1 x) = \frac{2}{L k_1^3} \underbrace{\int_{-\pi/2}^{\pi/2} dy\, y^2 \cos^2(y)}_{\frac{\pi^3 - 6\pi}{24}}$$

check

$$\langle \{x, p\} \rangle = 2 \mathrm{Re} \langle x p \rangle$$

$$\langle x p + p x \rangle = \langle x p \rangle + (\langle x p \rangle)^*$$

$$\langle x p \rangle = -i\hbar \int dx\, \psi(x) \frac{\partial \psi}{\partial x}$$

$\psi(x)$ reale

$$2 \mathrm{Re} \langle x p \rangle = 0$$

$$\boxed{\langle x^2(t) \rangle = \langle x^2(0) \rangle + \frac{\langle p^2(0) \rangle}{2m} t^2}$$

il pacchetto si allarga nel tempo