Esercises: set 2

1. Consider the following Hamiltonian H expressed i terms of an orthonormal basis $\{|n>,\ n = 1, \cdots 4\}$

$$H = E\left(|1><3| + |3><1| + |2><2| + |4><4|\right) .$$

Take the state $|s, t = 0 >= \frac{1}{\sqrt{3}}\left(|1> + |2> + |4>\right)$ as the initial state of the system at $t = 0$

- Determine the eigenvalues E_i and the eigenstates $|\lambda_i>$ of H.
- Determine the time evolution of the initial state at any time t.
- Compute the probability $P(\lambda_i)$ of measuring the values of energy λ_i on the state $|s, t>$ at the time t.

2. Consider a particle of mass m with a Hamiltonian

$$H = \frac{p^2}{2\,m} - \frac{p^4}{8\,m^3\,c^2} ;$$

where p is momentum operator and c is a constant.

- Solve the Heisenberg equation for the time evolution of the operators p_H and x_H in the Heisenberg picture.
- Take as initial state $|\psi, t = 0>$ at time $t = 0$ such that the density probability of finding the particle in the point x is a gaussian packet centered in x_0 with width σ.
- Determine the most general wave function of the system at $t = 0$.
- Knowing that $<\psi, t = 0|p|\psi, t = 0 >= k\,\hbar$, determine for any t the following averages

$$\begin{aligned} \bar{p}(t) &=< \psi, t|p|\psi, t > , \qquad & \Delta p^2(t) &=< \psi, t|p^2 - \bar{p}^2|\psi, t > , \\ \bar{x}(t) &=< \psi, t|p|\psi, t > , \qquad & \Delta x^2(t) &=< \psi, t|x^2 - \bar{x}^2|\psi, t > . \end{aligned}$$