Dipartimento di Scienze Fisiche e Chimiche

BACHELOR DEGREE IN PHYSICS

“INTRODUCTION TO THEORETICAL PHYSICS”

Teacher: Prof. Sergio Ciuchi ***CFU***: 12

Aim of the course

The goal of this course is to provide an introduction to the formalism of quantum mechanics and provide some basic notions of statistical mechanics.

On successful completion of this course the student should understand the fundamental concepts of quantum mechanics, quantum perturbation theory and basic atomic theory as well as the statistical approach to the equilibrium properties of classical and quantum perfect gases.

URL: http://www.aquila.infn.it/ciuchi/

Programme

1. **The formalism of Quantum Mechanics**

   Quantum states, the dual space and the scalar product . The Dirac formalism, orthonormal bases , representations of quantum states, relations of completeness.

   Linear operators matrix elements , self-adjoint operators, eigenvectors and eigenvalues of self-adjoint operators, degeneration, physical quantities, eigenstates of physical quantities, the physical meaning of the probability amplitudes, measurement of a physical quantity ,

   Coordinate operator, the eigenstates of the coordinate. Coordinate representation, calculation of matrix elements and scalar products in the coordinate representation.

   The momentum in Quantum Mechanics as a generator of infinitesimal translations

   Momentum operator, eigenfunctions. Momentum representation, calculation of matrix elements in the momentum representation, expression of the scalar products, relations with the coordinate representation.

   Group of translations, its commutativity , translation of the wave function and of the quantum state. Normalization of momentum eigenstates.

2. **Schrödinger's equation and the uncertainty principle**

   Hamiltonian as the generator of the infinitesimal classical motion, time-dependent Schrodinger equation and unitary time evolution. The energy representation, time-independent Schrodinger equation.

   The Gaussian wave-packet: expression of the wave function in the momentum and coordinate representation, temporal evolution , calculation of the variance of the distribution of the position in time, the product uncertainty , time-reversal invariance .

   Compatible observables. Common set of compatible observables, generalized uncertainty principle , commotation rules for canonical variables .

   Heisenberg representation: the equations of motion of the operators, the average values and the Ehrenfest theorem . Conservation of momentum and translational symmetry, applications to the evolution of the Gaussian wave-packet. Conserved quantities in Quantum Mechanics. Current probability, continuity equation, the group velocity of a wave-packet.

3. **Quantum harmonic oscillator and other one-dimensional problems**

   General properties of the solution of the one-dimensional time-independent Schodinger equation. Qualitative behavior of eigenstates in one dimension. Particle in an infinite potential well. Particle in a finite potential well: eigenstates and eigenvalues of the discrete spectrum, uncertainty estimate of the zero point energy.

   Potential step: reflection and transmission coefficients, the classical limit. Potential barrier tunneling and resonant scattering.

   Harmonic oscillator, the classical solution correspondence principle, creation and destruction operators, constructive derivation of the eigenstates and eigenvalues.

   Matrix elements of position and momentum, and their calculation. Time evolution in the Heisenberg formalism. Ehrenfest's theorem for the harmonic oscillator. Displaced harmonic oscillator eigenstates and eigenvalues.

4. **Angular momentum and spin.**

   Angular momentum as a generator of infinitesimal rotations and its expression in the coordinate representations. Commutation rules. Simultaneous eigenfunctions of L^2 and Lz, derivation of the spectrum. Matrix elements of L+, L-, Lx Ly. Expression of the angular momentum operators in spherical coordinates. Spherical harmonics, explicit calculations for l = 0, l = 1. P orbitals. Parity of the spherical harmonics.

   The spin. A charged particle of spin 1/2 in a static magnetic field. Orbital angular momentum and spin. Pauli spinors. Pauli matrices and their properties. Expression of a generic 2x2 matrix in terms of the Pauli matrices. Rotation of the spin. Two-level systems, temporal evolution. Spin precession.

Composition of two angular momenta, Clebsh-Gordan coefficients, construction of the eigenstates of J_1, J_2, J^2, J_z. Example: composition of two spin 1/2.

5. **Central potential and hydrogen atom.**

Schrödinger equation in the case of a central potential, the radial equation. Isotropic harmonic oscillator solution by separation of variables and in spherical coordinates.

The two-body problem, separation of coordinates. Separable Hamiltonians: form of the general solution. Hydrogen atom: atomic units and the radial equation.

Hydrogen atom solution for the radial part, eigenvalues, accidental degeneracy, diagram of the levels and their degeneracy, the ground state and the first excited states.

6. **Time-independent perturbation theory.**

Perturbation theory non-degenerate case. First and second order energy corrections and first-order correction to the states.

Degenerate spectrum: first-order correction to the energies and zeroth order for the states. Spontaneous breaking of symmetry (notes).

7. **Identical particles.**

Identical particles: exchange symmetry operator for two particles and its properties, bosons and fermions. Symmetric and antisymmetric wave functions. Slater determinant. Pauli principle.

Confined Fermions in dimensions d = 1,2,3: Fermi energy and its density dependence. Density of states.

8. **Basics of statistical mechanics.**

Maxwell distribution, distribution of position and momenta. , Liouville's theorem , the stationary distribution . Microcanonical distribution. Equiprobability and the Boltzmann formulation for the entropy, the case of the perfect gas. Thermodynamic relationships, the second principle, Gibbs paradox, correct Boltzmann counting. Maxwell distribution obtained from microcanonical distribution.

Energy distribution of independent. Canonical distribution: thermodynamic potential, energy fluctuations, De Broglie wavelength. Entropy and Sackur Tetrode formula, energy equipartition, ideal gas in the presence of an external field .

Open thermodynamic systems , Grandcanonical distribution, fluctuations in the number of particles.

Energy distribution of quantum particles, the partition function as a sum over the quantum states. Quantum ideal gases the case of Fermions and Bosons. Fermi gas at zero temperature. Black body.

Reference text books:
J.J. Sakurai "Modern quantum mechanics", Addison Wesley
C. Cohen Tannoudji et al.,"Quantum Mechanics", Vol **I-II**, John Wiley & Sons
P.A.M. Dirac "Principles of Quantum Mechanics", Oxford university press
K. Huang "Statistical Mechanics", John Wiley & Sons
Course notes

Assessment methods and criteria: Oral examination and 3 written tests during the course or a final written exam.