Exercise #3

due date: November 18th 2025

a) Consider two one particle states made by one-dimensional gaussians

$$g_1(x) = A\left(\exp(-\frac{(x-x_0)^2}{2\sigma^2})\right)^{1/2}$$
$$g_2(x) = A\left(\exp(-\frac{(x+x_0)^2}{2\sigma^2})\right)^{1/2}$$

with \$A\$ being a normalisation constant. Using these states write a possible state for 2 fermions and 2 bosons in state \$1\$ or \$2\$ neglecting the spin component. Calculate the average distance $< | x_1\text{-}x_2 |^2 >$ as a function of x_0. Comment the results. *Optional* Redo the same calculations for Boltzmann's particles.

b) Consider the perfect Fermi and Bose gas with a general single particle dispersion \$\epsilon\$(p)= a $|p|^b$ Determine:

- the density of the states (DOS)

$$N(\epsilon) = \frac{V}{h^3}\int d^3p\delta(\epsilon - \epsilon(p))$$

- the relation between PV and U as a function of the DOS parameters and verify that for massive non relativistic particles either bosonic or fermionic the relation is exactly the same as for Boltzmann's particles

c1) Consider a perfect gas of bosonic ultra-relativistic particles in 3 dimensions with vanishing chemical potential (black body). Calculate internal energy as a function of temperature and using the relation derived in the previous point prove that the radiation pressure is independent of volume and is proportional to T^4.

d1) Read the notes Prove that the equilibrium single-particle density matrix for Boltzmann particle

$$\hat{\rho} = \frac{\exp(-\beta H^{(1)})}{Z^{(1)}}$$

is for a free particle

$$\langle x|\exp(-\beta H)|y\rangle/Z^{(1)} = A\exp(-(x-y)^2/a\lambda^2)$$

where $Z^{(1)}$ is the single particle partition function, \lambda is the De Broglie wavelength and |x>,|y> position eigenstates (in 3 dimensions). Find the normalization (A) for a tree dimensional space and the numerical coefficient (a). Comment the results. Compare the result for < x| \rho |y > to that for <p_1| \rho |p_2 > with p_1 and p_2 are three dimensional wave vectors.

d2) *Optional* Consider the equilibrium single-particle density matrix in the grand-canonical ensemble for massive non-relativistic non-interacting Bose particles. Write its form in momentum and position (3d) representation. Comment the limit \mu → 0^- for the density matrix in the coordinate representation. Using the fugacity expansion to all order or the integral expression of the density matrix write a code to evaluate numerically it when \mu < 0.

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