Exercise #4

due date: December 2nd

a) In the mean field solution of the Ising model plot the isotherms $h(m)$ in the tree cases $T > T_c, T = T_c, T < T_c$. Plot the mean field estimate for the free energy $F(T, h)$ as a function of h in the tree cases $T > T_c, T = T_c, T < T_c$ and discuss the results. I advice to use *gnuplot* which allows to plot easily parametric functions.

b) Prove the isomorphism between the Ising model in the Canonincal Ensemble and the Lattice Gas model in the Grand-Canonical ensemble i.e. prove that $H_{LG} - \mu < N >$ maps onto H_{Ising} up to some additive constants. After the solution of the previous point you will be able to plot the isotherms in the pressure-volume space for the lattice gas and discuss the results.

In the following you can alternatively choose c1) or c2).

c1) Read the notes about the Kac ring model:

- which set of variables describes a *microscopic* state?
- which set of variables describes the *macroscopic* state?
- write a code (or ask ChatGPT to do it...) to calculate the number of black and white point (you can also do it considering a small size say N=4 and doing the evolution by hand...)
- compare the output of the code with the "molecular-chaos" solution given in the notes and discuss the results.

c2) Read the notes about the Logistic map:

- write a code (or ask ChatGPT to do it...) to calculate the map evolution and the Lyapunov exponent.
- discuss the results in the parameters space described in the note discussing the local stability and the Lyapunov exponent.

d) Read the chapter 3 of the textbook (Kerson Huang, "Statistical Mechanics", Second Edition, Wiley). Try to answer to one of the problems of the chapter at your convenience (or ask ChatGPT to do it...). If you dare you can try exercise 3.5 and for that it would be useful to read the notes on adiabatic expansion as well as this paper.