Exercise #5

due date: January 8th 2025

a) Following the lines of the notes derive the statistical properties of the noise term $\xi(t)$ as a result of the equilibrium distribution for the oscillators. Perform explicitly the $N\rightarrow \infty$ limit using the Ohmic spectral density.

b) Consider the stochastic equation for the moment of a particle under the action of external random forces $\xi(t)$ (in one dimension):

$\dot{p}(t)=-\gamma p(t) + \xi(t)$ + F

where

$\langle \xi(t) \rangle = 0$

$\langle \xi(t) \xi(t')\rangle = 2 M \gamma k_b T \delta(t-t')$

and F is a constant in space and time external force.

- calculate the average $<p(t)>$ in the presence of external force for all times and show that it tends to a constant value.
- in the absence of external force $F=0$ calculate $<p^2(t)>$ and show that it reaches the Maxwell-Boltzmann prediction for large times.

c) Read the notes and discuss how to derive the Fokker-Planck equation from the Langevin equation

d) The following point is optional

- in the absence of external force $F=0$ derive without approximation the average mean square displacement

$\Delta(t)=\langle |x(t)-x(0)|^2 \rangle$

- Derive the behaviour of $\Delta(t)$ for large and small times and define the time scale above which the behaviour of $\Delta(t)$ is **linear** in time.

Last updated 2025-12-12 10:08:24 CET