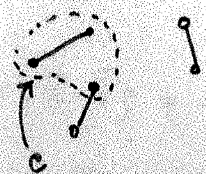
Liouville $\Longrightarrow$ BBGKY (vedere Reichl)

$$\frac{\partial \rho}{\partial t} = -\{\rho, H\} = \frac{\partial H}{\partial q}\frac{\partial \rho}{\partial p} - \frac{\partial H}{\partial p}\frac{\partial \rho}{\partial q}$$

$$H = \underbrace{H_0}_{\text{e cinetica}} + H_{int} =$$

$$= \underbrace{H_0^{(c)} + \overbrace{H_0^{(\bar{c})} + H_{int}^{(c)}}^{H^{(\bar{c})}}}_{H^{(c)}} + H_{int}^{(\bar{c})} + H_{int}^{(c\bar{c})}$$



$\boxed{\bullet \in c}$ ← cluster

$\boxed{\circ \in \bar{c} = 1 - c}$ ← environment ("bagno"...)

$$\frac{\partial \rho}{\partial t} = \{H^{(c)}, \rho\} + \quad \leftarrow \text{SISTEMA}$$

$$+ \{H^{(\bar{c})}, \rho\} + \quad \leftarrow \text{BAGNO}$$

$$\{H_{int}^{(c\bar{c})}, \rho\} \quad \leftarrow \text{INTERAZIONE}$$

$$\rho^{(s)} = \operatorname*{tr}_{(\bar{c})} \rho = \int \prod_{s+1}^{N} dp_\lambda^{3N} \, dq_\lambda^{3N} \; \rho(q_1 \ldots q_s \; q_{s+1} \ldots q_N, p_1 \ldots p_s \; p_{s+1} \ldots p_N)$$

$$\operatorname*{tr}_{(\bar{c})} \frac{\partial \rho}{\partial t} = \frac{\partial}{\partial t} \operatorname{tr} \rho = \frac{\partial}{\partial t} \rho^{(s)}$$

$$\frac{\partial \rho^{(s)}}{\partial t} = \underset{(1+2)}{\{H^{(c)}, \operatorname{tr} \rho\}} + \underset{(A+B)}{\cancel{\operatorname*{tr}_{(\bar{c})} \{H^{(\bar{c})}, \rho\}}} + \underset{(C)}{\operatorname*{tr}_{(\bar{c})} \{H_{int}^{(c\bar{c})}, \rho\}}$$

↑ dimostrazione dopo

$$\operatorname*{tr}_{(\bar c)}\Big(\underset{(a)}{\frac{\partial H^{(\bar c)}}{\partial \bar q}\frac{\partial \rho}{\partial \bar p}} - \underset{(b)}{\frac{\partial H^{(\bar c)}}{\partial \bar p}\frac{\partial \rho}{\partial \bar q}}\Big)$$

<u>GAUSS GENERALIZZATO</u>

$$\int d^d x \; \vec\nabla \rho(x) \cdot \vec a(x) = \sum_1^d{}_i \int \Big(\prod_1^d{}_k dx_k\Big)\Big(\frac{\partial \rho}{\partial x_i}\Big) \cdot a_i =$$

$$= \sum_1^d{}_i \int \Big(\prod_{k\neq i} dx_k\Big)\Big\{ \cancel{a_i(x)\,\rho(x)\Big|_{x_i\to-\infty}^{x_i\to+\infty}} - \int dx_i \, \rho \frac{\partial}{\partial x_i} a_i \Big\}$$

$\rho(q\to\infty)=0$

$\rho(p\to\infty)=0$

$$\int d^d x \; \vec a \cdot \vec\nabla \rho = -\int d^d x \; \rho \, \vec\nabla \cdot \vec a$$

a) $$\int d\bar p \, d\bar q \Big(\underbrace{\frac{\partial H^{(\bar c)}}{\partial \bar q}}_{\vec a}\underbrace{\frac{\partial \rho}{\partial \bar p}}_{\nabla_{\bar p}\rho}\Big) = -\int d\bar q \int d\bar p \; \rho \frac{\partial}{\partial \bar p \, \partial \bar q} H^{(\bar c)}$$

b) $$\int d\bar q \int d\bar p \, \frac{\partial H}{\partial \bar p}\frac{\partial \rho}{\partial \bar q} = -\int d\bar p \int d\bar q \; \rho \frac{\partial}{\partial \bar q \, \partial \bar p} H^{(c)}$$

$= 0$ CVD

$$\frac{\partial \rho^{(s)}}{\partial t} = \{H^{(c)}, \rho^{(s)}\} + \operatorname*{tr}_{(\bar c)} \{H^{(c\bar c)}_{int}, \rho^{(N)}\}$$

Se non c'è interazione $\rho^{(s)}$ soddisf. Liouville $\forall S$!!

$$(C) = \int d\bar{p}\, d\bar{q} \left\{ \frac{\partial H_{int}^{(c\bar{c})}}{\partial \bar{q}} \frac{\partial \rho^{(N)}}{\partial \bar{p}} - \cancel{\frac{\partial H_{int}^{(c\bar{c})}}{\partial \bar{p}} \frac{\partial \rho^{(N)}}{\partial \bar{q}}} + \right.$$

(H_{int} dip. solo dalle q)

$$\left. + \frac{\partial H_{int}}{\partial q} \frac{\partial \rho^{(N)}}{\partial p} - \cancel{\frac{\partial H_{int}}{\partial p} \frac{\partial \rho^{(N)}}{\partial q}} \right\} =$$

$$= \int \prod_{s+1}^{N} d^{3N}p_k\, d^{3N}q_k \, \Sigma_i^{(c)} \Sigma_j^{(\bar{c})} \Big(\underbrace{\frac{\partial V_{ij}}{\partial q_j} \frac{\partial \rho}{\partial p_j}}_{\in \bar{c}} + \underbrace{\frac{\partial V_{ij}}{\partial q_i} \frac{\partial \rho}{\partial p_i}}_{\in c} \Big) =$$

$$= \int \prod_{s+1}^{N} d^{3N}p_k\, d^{3N}q_k \, \Sigma_i^{(c)} \Sigma_j^{(\bar{c})} \hat{\Theta}_{ij}\, \rho^{(N)} =$$

integrate

$$= \Sigma_i^{(c)} \int \prod_{s+1}^{N} d^{3N}p_k\, d^{3N}q_k \, (N-s)\, \hat{\Theta}_{i\,s+1}\, \rho^{(N)} =$$

una qualsiasi

$$= \Sigma_i^{(c)} \int d^{3N}p_{s+1}\, d^{3N}q_{s+1}\, (N-s)\, \hat{\Theta}_{i\,s+1}\, \rho^{(s+1)}$$

$$\underbrace{\frac{\partial \rho^{(s)}}{\partial t} + \{\rho^{(s)}, H\}}_{\text{Liouville}} = \underbrace{\Sigma_i^{(c)} (N-s) \int d^{3N}p_{s+1}\, d^{3N}q_{s+1}\, \hat{\Theta}_{i\,s+1}\, \rho^{(s+1)}}_{\text{Forzante}}$$

$$\hat{\Theta}_{i\,s+1}\, \rho^{(s+1)} = \frac{\partial V_{i\,s+1}}{\partial q_i} \frac{\partial \rho^{(s+1)}}{\partial p_i} + \frac{\partial V_{i\,s+1}}{\partial q_{s+1}} \frac{\partial \rho}{\partial p_{s+1}}$$

$$\int d^{3N}p_{s+1}\, d^{3N}q_{s+1}\, \frac{\partial V_{i\,s+1}}{\partial q_{s+1}} \frac{\partial \rho}{\partial p_{s+1}} = - \cancel{\int d^{3N}p_{s+1}\, d^{3N}q_{s+1}\, \rho \frac{\partial}{\partial p_{s+1}} \frac{\partial V}{\partial q_{s+1}}} = 0$$

$$\Sigma_i^{(c)} \int d^{3N}p_{s+1}\, d^{3N}q_{s+1}\, \hat{\Theta}_{i\,s+1}\, \rho^{(s+1)} = - \Sigma_i^{(c)} \int d^{3N}p_{s+1}\, d^{3N}q_{s+1}\, \vec{F}_{i\,s+1} \cdot \frac{\partial \rho^{(s+1)}}{\partial \vec{p}_i}$$