EIGENSTATE THERMALIZATION HYPOTESIS (ETH)

$$\overline{\langle \hat{A}(t) \rangle}_T = \sum_{\alpha\beta} c_\alpha^* c_\beta \frac{e^{i\omega_{\alpha\beta}T}-1}{i\omega_{\alpha\beta}T} \langle \alpha|\hat{A}|\beta\rangle =$$

$$= \sum_{\alpha} |c_\alpha|^2 \langle \alpha|\hat{A}|\alpha\rangle + \sum_{\alpha\neq\beta} c_\alpha^* c_\beta \underbrace{\frac{e^{i\omega_{\alpha\beta}T}-1}{i\omega_{\alpha\beta}T}}_{\text{DEPHASING}} \underbrace{\langle \alpha|\hat{A}|\beta\rangle}_{\text{EIGENSTATE THERMALIZATION HYPOTESIS!!}}$$

DEPHASING
↓
GOOD x SMALL # OF PARTICLES

IF N PARTICLES
$\omega_{\alpha\beta} \sim e^{-N} \xrightarrow[N\to\infty]{} \emptyset$

$\Rightarrow T \sim \frac{1}{\omega_{\alpha\beta}} \xrightarrow[N\to\infty]{} \infty$

T: DEPHASING TIME

ETH: $\langle \alpha|\hat{A}|\beta\rangle = A_{\alpha\beta}(E_{\alpha\beta})\,\delta_{\alpha\beta} + f_A(E_{\alpha\beta},\omega_{\alpha\beta})\,\Omega^{-1/2}(E_{\alpha\beta})\,R_{\alpha\beta}$

$\Omega^{-1/2} \to e^{-\frac{S(E)}{2}} \sim e^{-N} \to \emptyset$

↓

PAPERS:
- VON NEUMANN QUANTUM ERGODIC THEORY (1929)
- SREDNICKI CHAOS & QUANTUM THERMALIZATION (1994)

IF $\hat{A}$ MOVES FEW PARTICLES (LOCAL ACTION IN SPECTRUM)

→ HYPOTESIS ON OBSERVABLES !!

⇒ TO HAVE ETH:

- LOCAL OBSERVABLES
- $N \to \infty$!!!

NOTE:

$\omega_{\alpha\beta} \sim e^{-N} \Rightarrow$ CAN $\omega_{\alpha\beta}$ REMOVE THE ACTION OF $\langle\alpha|\hat{A}|\beta\rangle$ BECAUSE $\omega_{\alpha\beta}^{-1}\langle\alpha|\hat{A}|\beta\rangle$ IN SUMMATION $\sum_{\alpha\neq\beta}$?

NO! BECAUSE OF $e^{i\omega_{\alpha\beta}T}-1$

IN FACT:

$\omega_{\alpha\beta}T \ll 1 \Rightarrow T \ll e^N$ ⇒ PHYSICAL TIMES !!

$$\sum_{\alpha\neq\beta} c_\alpha^* c_\beta \frac{e^{i\omega_{\alpha\beta}T}-1}{i\omega_{\alpha\beta}T}\langle\alpha|\hat{A}|\beta\rangle \simeq$$

$$\simeq \sum_{\alpha\neq\beta} c_\alpha^* c_\beta \frac{(i\omega_{\alpha\beta}T)\langle\alpha|\hat{A}|\beta\rangle}{i\omega_{\alpha\beta}T} \simeq$$

$$\simeq \sum_{\alpha\neq\beta} c_\alpha^* c_\beta \langle\alpha|\hat{A}|\beta\rangle \xrightarrow[N\to\infty]{\text{AS}} 0$$

IF ETH HOLDS → NO NEED OF TEMPORAL AVERAGE

$\langle \psi(t)|\hat{A}|\psi(t)\rangle \sim \langle \hat{A}\rangle_{\rho \to \text{THERMAL}}$ x ALMOST ALL TIMES

IN PARTICULAR $\forall\, |\alpha\rangle$

$\langle \alpha|\hat{A}|\alpha\rangle \sim \langle\hat{A}\rangle_\rho$ → STRONG CONDITION

WEAK ETH → $\exists$ SCAR STATES
↓
ATHERMAL STATES → $\langle\alpha|\hat{A}|\alpha\rangle \neq \langle\hat{A}\rangle_\rho$

ONLY A SMALL FRACTION OF $|\alpha\rangle$ VIOLATES ETH

→ $\langle\psi(t)|\hat{A}|\psi(t)\rangle \sim \langle A\rangle_\rho$ x ALMOST ALL TIMES

→ NORMAL TYPICALITY !

BUT QUANTUM THERMALIZATION STILL OCCURS

→ x THIS ETH

QUANTUM CHAOS & ETH
(NON-INTEGRABILITY)

$$\langle\alpha|\hat{A}|\beta\rangle = A_{\alpha\beta}(E_{\alpha\beta})\,\delta_{\alpha\beta} + f_A(E_{\alpha\beta},\omega_{\alpha\beta})\,\Omega^{-1/2}(E_{\alpha\beta})\,\boxed{R_{\alpha\beta}}$$

$R_{\alpha\beta}$ → RANDOM NUMBER

MATH OF RANDOM MATRICES IS INSIDE ETH

THIS COMES FROM DETERMINISTIC CHAOS: "EFFECTIVE STOCHASTICITY CONFERRED BY A DETERMINISTIC SYSTEM"

AT THE STATE OF THE ART THE BIGGEST PART OF SCIENTISTS THINKS

THAT QUANTUM ERGODICITY IS RELATED TO CHAOS (QUANTUM)
$\rightarrow$ (IN PHYSICAL TIMES)

<u>BUT</u> IN CLASSICAL PHYSICS ERGODICITY $\not\Leftrightarrow$ CHAOS
(EVEN IF FOR A LONG TIME THE OPPOSITE WAS THOUGHT)
ERGODICITY IS A MATTER OF # OF PARTICLES 'N' AND
OF THE OBSERVABLE $\rightarrow$ NO ROLE OF CHAOS!!

IN QUANTUM MECHANICS? $\longrightarrow$ OPEN QUESTION!!

ALTERNATIVE FORMULATION OF ETH

IN GENERAL IF ENSEMBLE EQUIVALENCE HOLDS $\langle \hat{H} \rangle_{mc} \sim \langle \hat{H} \rangle_{can}$

<u>IF</u> ETH HOLDS $\underbrace{\langle m | \hat{H} | m \rangle}_{E_m} \sim \langle \hat{H} \rangle_{can}$ $\rightarrow$ WITH APPROPRIATE CHOICE OF β

$E_m \longrightarrow$ IT IS NOT $\langle \hat{H} \rangle_{mc}$

$\forall\ |m\rangle$ IT IS FOR A SINGLE EIGENSTATE!!!