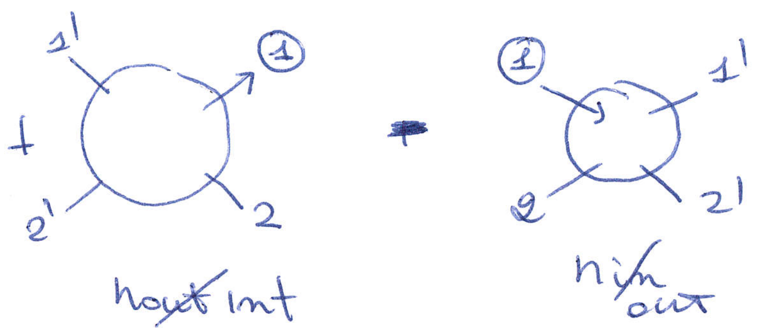


Boltzmann Equation H-theorem

$$dn_{\cancel{out}\, in} = \int_{1'2'2} d1'd2' f(1'2')\, W_{1'2'\to 12}\, d1\, d2$$

$$dn_{\cancel{in}\, out} = \int_{1'2'2} d1\, d2\, f(12)\, W_{12\to 1'2'}\, d1'd2'$$

$$dn_{out} = d1\, n_{out}$$
$$dn_{in} = d1\, n_{in}$$

$$\frac{\partial f(1,t)}{\partial t} = \int d1'd2'd2 \left(f(12) W_{12\to 1'2'} - f(1'2') W_{1'2'\to 12} \right)$$

Symmetry $W_{12\to 1'2'} = W_{1'2'\to 12}$ Parity / Time Reversal

~~Stossen~~ Molecular Chaos $f(12) = f(1) f(2)$

$$\frac{\partial f(1,t)}{\partial t} = \int d1'd2'd2 \left(f(1') f(2') - f(1) f(2) \right) W_{12\to 1'2'}$$

~~Stross~~

Stoss zahl ansatz

shock

number
numerous
many

Cross Sections

differential transition probability

$$dP_{if} = W_{i\to f}\, df$$

transition rates

initial states

final states

volume of final states

$$dP_{if} = I_i \, d\sigma_{if}$$

flux of incoming particles

differential cross section

$$dP_{if} = df\, \delta^{(4)}(p_f - p_i)\, |T_{fi}|^2$$

$W_{i\to f}$

T-matrix

$$\delta^{(4)}(p_f - p_i) = \delta(\vec{p}_1{}' + \vec{p}_2{}' - \vec{p}_1 - \vec{p}_2)\,\delta(p_1'^2 + p_2'^2 - p_1^2 - p_2^2)$$

momentum conservation

energy conservation

non relativistic momentum + energy conservation in the Boltzmann eq the cross Sections appears integrated

$$\int \cancel{d^3p_1'}\, d^3p_2'\, d^3p_2 \quad W_{12\to 1'2'} \, (\dots)$$

momentum conservation

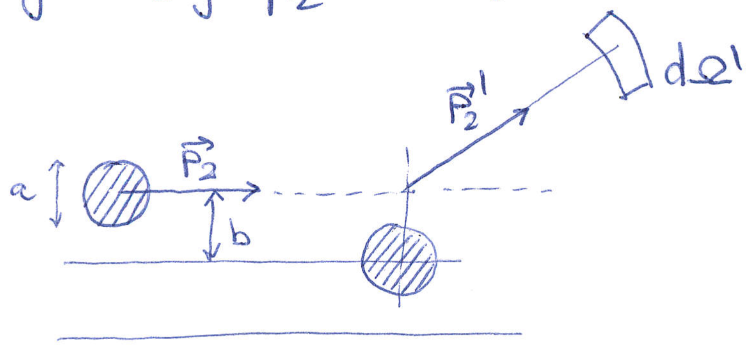
then we remain with

$$\int d^3p_2' \int d^3p_2 \,(\dots)$$

$$\cancel{\int_0^\infty dp_2'} \int_0^{2\pi} d\varphi' \int_0^{\pi} d\theta \; p_2'^2 \sin\theta \int d^3p_2 \,(\dots)$$

energy conservation kills the dp_2' integral

$$\int d\Omega_2' \int d^3p_2 \,(\dots)$$



in terms of velocities

$$\frac{\partial f(\vec{v}_1,t)}{\partial t} = \frac{a^2}{4} \int d\Omega' \int d^3\vec{v}_2 \, |\vec{v}_1 - \vec{v}_2| \left[f^{(2)}(v_2', v_1') - f^{(2)}(v_2, v_1) \right]$$

cfr Huang

H-theorem

$$\frac{\partial f(1)}{\partial t} = 0 \iff f(1)f(2) = f(1')f(2')$$

valid when $W_{12\to 1'2'} \neq 0$
i.e. when Momentum + Energy is conserved

H-theorem

if f solution of B.E. then the quantity

$$H(t) = \int d^3p\, f^{(1)}(p,t) \ln f^{(1)}(p,t)$$

is a <u>monotonic</u> <u>decreasing</u> function $\frac{dH}{dt} \leq 0$

Consequences

System tends to equilibrium and $f(\vec{p},t) \to e^{-\beta \frac{\vec{p}^2}{2m}} A$

maxwell-boltzmann distribution is the equilibrium long-time distribution of the non-equilibrium B.E.

proof of H-theorem

$$\dot{H} = \int d^3\vec{p} \left(\dot{f} \ln f + \frac{f}{f}\dot{f}\right) = \int d^3p\, \dot{f}(\ln f + 1)$$

$$\dot{H} = \int d1\, d2\, d1'\, d2'\, W_{12\to 1'2'} \left(f(1')f(2') - f(1)f(2)\right)\left(\ln f(1) + 1\right)$$

SYMMETRIZATION

~~(IN) ⇄ (OUT)~~ $1 \rightleftarrows 2$

$$\dot{H} = \int d2\, d1\, d1'\, d2'\, W_{21\to 1'2'} \left(f(1')f(2') - f(2)f(1)\right)\left(\ln f(2) + 1\right)$$

$$\dot{H} = \int d1\,d2\,d1'd2'\, W_{12\to 1'2'} \left(f(1')f(2') - f(1)f(2) \right) \frac{1}{2} \left(\ln f(1)f(2) + 2 \right)$$

$12 \rightleftarrows 1'2'$

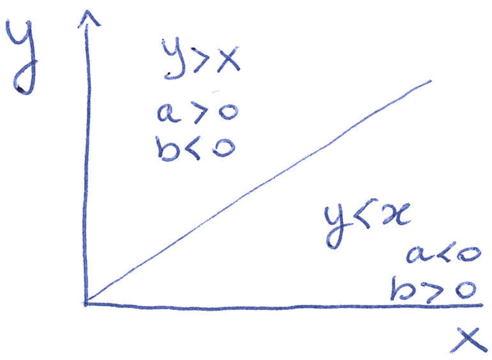
$$\dot{H} = \int d1'd2'd1\,d2\, W_{1'2'\to 12} \left(f(1)f(2) - f(1')f(2') \right) \frac{1}{2} \left(\ln f(1')f(2') + 2 \right)$$

$$\dot{H} = \int d1'd2'd1\,d2\, W_{12\to 1'2'} \underbrace{\left(f(1')f(2') - f(1)f(2) \right) \frac{1}{4} \left(\ln f(1)f(2) - \ln f(1')f(2') \right)}$$

$x = f(1)f(2) \qquad y = f(1')f(2')$

$$\underbrace{(y-x)}_{(a)}\underbrace{(\ln x - \ln y)}_{(b)} = f(x,y)$$

with $x, y > 0$



$\Rightarrow f(x,y) \leq 0$

$f(x,y) = 0 \Leftrightarrow x = y$

where $W_{12\to1'2'} \neq 0 \Rightarrow$ momentum + Energy conserved

$$\begin{cases} \vec{p_1}+\vec{p_2} = \vec{p_1}'+\vec{p_2}' \\ p_1^2+p_2^2 = p_1'^2+p_2'^2 \end{cases}$$

then

$$f(\vec{p_1})f(\vec{p_2}) = f(\vec{p_1}')f(\vec{p_2}')$$

take the logrorithm

$$\ln f(1) + \ln f(2) = \ln f(1') + \ln f(2')$$

extra conservation low $\Rightarrow$ Not indep

$$\ln \underbrace{f(\vec{p})}_{\text{scalar}} = A + B|\vec{p}|^2$$

$$f(p) = A e^{-|\vec{p}|^2/2\sigma^2}$$

$A = \dfrac{1}{(\sqrt{2\pi\sigma^2})^3}$ normalizacio

$\sigma =$? $\Leftarrow$ equilibrium

~~perfect gas~~

Velocity & positions uncorrelated for a classical gas (liquid solid!!)

We want the average kinetic Energy to be the classical prediction

$$K = \frac{3}{2} N k_B T$$

$$K = \sum_1^N {}_i \left\langle \frac{|\vec{p}_i|^2}{2m} \right\rangle = \sum_1^N {}_i \sum_1^3 {}_\alpha \frac{\langle p_{i,\alpha}^2 \rangle}{2m} = \frac{3}{2} N \frac{\sigma^2}{m}$$

σ^2

$$\cancel{\frac{3}{2}}\cancel{N} \frac{\sigma^2}{m} = \cancel{\frac{3}{2}}\cancel{N} k_B T \qquad \sigma^2 = m k_B T$$

$$f(\vec{p}) = \frac{1}{(\sqrt{2\pi\sigma^2})^3} e^{-\frac{p^2}{2m k_B T}} = (\ldots) e^{-\beta\frac{p^2}{2m}}$$

M/B distribution

Stress Not the Perfect Gas!