Time/Space scaling BBGKY

$$\frac{\partial \rho}{\partial t} = \{H,\rho\} = \underbrace{\left[\frac{\partial H}{\partial q}\frac{\partial}{\partial p} - \frac{\partial H}{\partial p}\frac{\partial}{\partial q}\right]}\rho$$

physical units $\frac{[\rho]}{T}$

physical units $\cancel{[\rho]}\ 1/T$

Diffrent terms in the r.h.s. gives different time-scales.

for $s=1$ $\rho^{(1)}(1,t)$ means $\rho(\bar{q}_1,\bar{p}_1,t)$

$$\frac{\partial \rho^{(1)}}{\partial t} = \left(\frac{\partial \phi}{\partial q_1}\frac{\partial}{\partial p_1} - \frac{\vec{p}_1}{m}\frac{\partial}{\partial \vec{q}_1}\right)\rho^{(1)} + \begin{pmatrix}\text{collision}\\ \text{term}\end{pmatrix}$$

$\hat{\mathcal{L}}_e$ $\hat{\mathcal{L}}_s$

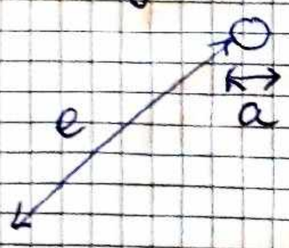
$$-(N-1)\int d2\, F^{int}(12)\,\frac{\partial}{\partial \vec{p}_1}\rho^{(2)}(12,t)$$

$d^3p_2\, d^3q_2$

associated to lenght scale of External confining potential l_e

associated to spatial variation of $\rho^{(1)}$ l_s

lenght scale are associated to typical particle velocity if density is sufficiently smalle



l interparticle distance

a radius of one particle

low density means

$$\ell \gg a \quad \text{with } \ell = \left(V/N\right)^{1/3} \rightarrow \text{associated to confining potential}$$

this means that at low density particles after interaction moves freely for a large spatial extent (the mean free path) with a typical velocity $\bar{v}$

therefore to each length scale we associate a time-scale

$$\ell_s \longrightarrow \tau_s = \ell_s/\bar{v}$$

$$\ell_e \longrightarrow \tau_e \approx \ell_e/\bar{v}$$

for a low density system not so far equilibrium $\ell_s \sim \ell_e$ this comes from equilibrium considerations

$$\underset{\uparrow}{\rho_{eq}}(pq) \propto e^{-\beta \frac{p^2}{2m}} \underbrace{e^{-\beta\phi(q)}}_{\downarrow}$$

$\rho^{(1)}$ length scale ℓ_s potential length-scale ℓ_e

and both can be approximated by ℓ

$$\ell_e \gtrsim \ell_s \sim \ell$$

at non-equilibrium sound waves creates spatial variation of $\rho^{(1)}$ which can be shorter than the length associated to confining potential however ...

... at low density

$$\boxed{\begin{matrix} l_e \gtrsim l_s \\ \tau_e \gtrsim \tau_s \end{matrix}} \gg \boxed{\begin{matrix} a \\ \tau_c \end{matrix}}$$

with τ_c the time associated to collisions

consider the scales associated with collision term

$$-(N-1)\int d^3p_2\, d^3q_2\; \vec{F}^{int}(12)\, \frac{\partial}{\partial \vec{p}_1}\, \rho^{(2)}$$

collision operator
has the physical dimension
of $1/T \Rightarrow 1/\tau_c$

Comparison of order of magnitude of different terms in BBGKY $s=1$

dimensions of $\rho^{(1)}$: by normalisation

$$\int d^3p\, d^3q\, \rho^{(1)} = 1 \qquad [\rho^{(1)}] = \frac{1}{(\text{action})^3}$$

dimensions of $\rho^{(2)}$

$$\int d^3p\, d^3q\, d^3p'\, d^3q'\, \rho^{(2)}(q q' p p' t) = 1$$

$$[\rho^{(2)}] = \frac{1}{(\text{action})^6} = [\rho^{(1)}]^2$$

$$\frac{\partial \rho^{(1)}}{\partial t} = \underbrace{\hat{\mathcal{L}}_e \rho^{(1)}}_{\frac{1}{\tau_e}[\rho^{(1)}]} + \underbrace{\hat{\mathcal{L}}_s \rho^{(1)}}_{\frac{1}{\tau_s}[\rho^{(1)}]} + \left(\begin{array}{c}\text{collision}\\ \text{term}\end{array}\right)$$

$$-(N-1)\int d^3p_2\, d^3q_2\, \underbrace{\vec{F}^{\,int}_{(12)} \cdot \frac{\partial}{\partial \vec{p}_1}}_{\frac{1}{\tau_e}}\, \underbrace{\rho^{(2)}}_{[\rho^{(1)}]^2}$$

extend to q_2
such that $F(12) \neq 0$
i.e to a volume a^3

At low density and near equilibrium state

$[\rho^{(1)}] = \frac{1}{V V_p}$ where V_p is a scale of dimensions p^3

$[\rho^{(2)}] = \frac{1}{V^2 V_p^2}$

the scale of the collision term is thus

$$N \cancel{V_p}\, a^3 \frac{1}{\tau_e} \frac{1}{V^2 V_p^{\cancel{2}}} = \frac{1}{V V_p}\, \rho a^3 \frac{1}{\tau_c}$$

$$= [\rho^{(1)}]\, \rho a^3 \frac{1}{\tau_c}$$

while $\tau_c \ll \tau_e, \tau_f$ the scale of collision term is not so much larger than the scale of $\hat{\mathcal{L}}_e \rho$ and $\vec{\mathcal{L}}_f \rho$ because of

$$\rho a^3 = \left(\frac{a}{\ell}\right)^3 \ll 1$$

therefore we must retain the collision term in $\rho^{(1)}$

Comparison of order of magnitude of different terms in BBGKY $s=2$

at variance with $s=1$ we have a term induced by interaction in the Liouvillian

$$\frac{\partial \rho^{(2)}}{\partial t} = \{H^{(2)}, \rho^{(2)}\} + \begin{pmatrix} \text{collision} \\ \text{term} \end{pmatrix} \text{ for } \rho^{(2)}(1,2,t)$$

$1 \to p_1 q_1$, $2 \to p_2 q_2$

$$(a) \quad \boxed{\left[\mathcal{L}_e^{(1)} + \mathcal{L}_e^{(2)} + \mathcal{L}_f^{(1)} + \mathcal{L}_f^{(2)}\right]\rho^{(2)} +}$$

$$(b) \quad \boxed{+\left(\vec{F}^{int}(12)\frac{\partial}{\partial \vec{p}_2} + \vec{F}^{int}(21)\frac{\partial}{\partial \vec{p}_1}\right)\rho^{(2)}}$$

while the blue term has the same scale as in $\rho^{(1)}$

$$(a) \quad \boxed{\frac{1}{\tau_e}[\rho^{(2)}] + \frac{1}{\tau_f}[\rho^{(2)}] \sim \frac{1}{\tau_e}[\rho^{(2)}]}$$

The red term is

$$(b) \quad \boxed{\frac{1}{\tau_c}[\rho^{(2)}]}$$

the collision term is now

$$-(N-2)\int d^3p_3\, d^3q_3 \left(F(13)\frac{\partial}{\partial \vec{p}_1} + F(23)\frac{\partial}{\partial \vec{p}_2}\right)\rho^{(3)}$$

$$[\rho^{(3)}] \sim [\rho^{(2)}]\frac{1}{V V_p}$$

therefore the size of this term is

$$\text{ⓒ}\quad N V_p a^3 \frac{1}{\tau_c}\frac{1}{V V_p}[\rho^{(2)}]$$

the r.h.s. of BBGKY for $s=2$ has the following order of magnitude

ⓐ $\frac{1}{\tau_e}[\rho^{(2)}]$

ⓑ $\frac{1}{\tau_c}[\rho^{(2)}]$

ⓒ $\rho a^3 \frac{1}{\tau_e}[\rho^{(2)}]$

and the collision term can be neglected w.r.t. ⓑ

~~Since the time~~ The leading term in $\rho^{(2)}$ is ⓑ which not only rules the time-scale for $\rho^{(2)}$ but also the space-scale

In ⓑ term the $F^{int}_{(12)}$ factors are $\neq 0$ only on a length scale $\ell_c \simeq a$ therefore the length scale for $\rho^{(2)}$ is a

this is in agreement with equilibrium calculation in gases where the spatial scale for $\rho^{(2)}$ is that of $g^{(2)}(r)$ (momenta factorise for classical systems) and the variation of $g^{(2)}(r)$ is on

$$\bar{r} = |\bar{q}_1 - \bar{q}_2| \simeq a$$

on the other hand on the spatial scale in which $\rho^{(1)}$ varies i.e. the position and momenta ~~deco~~ becomes independent btw different particles

$$\lim_{|\bar{q}_1-\bar{q}_2|\gg a} \rho^{(2)}(q_1 p_1\, q_2 p_2\, t) \simeq \rho^{(1)}(q_1 p_1 t)\, \rho^{(1)}(q_2 p_2 t)$$

Boltzmann's
Stosszahlansatz
Molecular Chaos