TRASF. DI LEGENDRE È CAMBIAMENTO
insieme statistico

$W(A)$: $\frac{\partial W}{\partial A} = \bar{A}$ $\rightarrow W = -\frac{1}{\beta}\ln Z$ controlo A

$$\overline{W}(\bar{A}) = W(A) - A\bar{A}$$

$$\frac{\partial \overline{W}}{\partial \bar{A}} = -A \rightarrow \overline{W} = -\frac{1}{\beta}\ln \overline{Z} \quad \text{controlo } \bar{A}$$

$$\overline{Z}(\bar{A}) = \int dA\, e^{-\beta[W(A) - A\bar{A}]} =$$

$$= \int dA\, e^{\beta A\bar{A}} Z(A) \quad \text{trasfo Laplace } Z$$

$$\frac{\partial}{\partial \bar{A}}\left[-\frac{1}{\beta}\ln \overline{Z}\right] = -\frac{1}{\beta}\frac{\partial}{\partial \bar{A}}\ln \overline{Z} = -\frac{1}{\beta}\left\{\frac{1}{\overline{Z}}\frac{\partial}{\partial \bar{A}}\overline{Z}\right\} =$$

$$= -\frac{1}{\beta}\frac{1}{\overline{Z}}\beta\int dA\, e^{-\beta[W(A) - \bar{A}A]} A = -\langle A\rangle_{\bar{A}}$$

media sulla distribuzione $\rho_{\bar{A}}(A) = \dfrac{e^{-\beta[W(A) - \bar{A}A]}}{\int dA\, e^{-\beta[W(A) - A\bar{A}]}}$

che ha $\bar{A}$ fix!

distribuzione di A ad $\bar{A}$ fix

Metodo punto sella

$$\overline{Z}(\bar{A}) = \int dA\, e^{-\beta\underbrace{[W(A) - A\bar{A}]}_{\text{estensivo}}} \simeq e^{-\beta[W(A^*) - A^*\bar{A}]}\int dA\, e^{-\beta\underbrace{\left[\frac{\partial^2}{\partial A^2}[W(A) - A\bar{A}]\right]_{A^*}}_{1/\beta\sigma_A^2}\frac{(A-A^*)^2}{2}}$$

$$\simeq e^{-\beta[W(A^*) - A^*\bar{A}]}\left(2\pi\sigma_A^2\right)^{1/2}$$

$\rightarrow$ A intensivo $\sigma_A \sim 1/\sqrt{N}$

$\rightarrow$ A estensivo $\sigma_A \sim \sqrt{N}$

da questo nel limite termodinamico

$$\boxed{W(A^*) - A^*\bar{A} = \overline{W}(\bar{A})} \quad \boxed{\beta\sigma_A^2 = 1/\left(\frac{\partial \bar{A}}{\partial A}\right)_{A^*}}$$

fluttuazione variabile fluttuante

$$\bar{Z}(\bar{A}) = \int dA \, e^{+\beta A\bar{A}} Z(A)$$

$$\boxed{\rho_{\bar{A}}(A) = \frac{e^{\beta A\bar{A}} Z(A)}{\bar{Z}(\bar{A})}}$$

distribuzione della variabile fluttuante A

Col metodo del punto sella

$$\underbrace{W(A) - A\bar{A}}_{\text{estensivo}} = \underbrace{W(A^*) - A^*\bar{A}}_{(0)} + \underbrace{\frac{1}{2}\left[\frac{\partial^2}{\partial A}(W(A) - A\bar{A})\right]_{A^*}(A-A^*)^2}_{(2)} +$$

$$+ \underbrace{\frac{1}{3!}\left(\frac{\partial^3 (W(A) - A\bar{A})}{\partial A}\right)_{A^*}(A-A^*)^3}_{(3)} + \mathcal{O}((A-A^*)^4)$$

$$\left(\frac{\partial^2 (W(A) - A\bar{A})}{\partial A^2}\right)_{A^*} = \frac{\partial}{\partial A}\left(\left(\frac{\partial W}{\partial A}\right) - \bar{A}\right)_{A^*} = \left(\frac{\partial \bar{A}}{\partial A}\right)_{A^*} = \frac{1}{\sigma_A^2}$$

$$\frac{\partial^3}{\partial A^3}(W(A) - A\bar{A}) = \left(\frac{\partial^2 \bar{A}}{\partial A^2}\right)_{A^*}$$

se A intensivo $\rightarrow$ $\bar{A}$ estensivo $\rightarrow \mathcal{O}(N)$

se A estensivo $\rightarrow$ $\bar{A}$ intensivo $\rightarrow \mathcal{O}\left(\frac{1}{N^2}\right)$

Il termine (3) è dunque di ordine

A intensivo $\longrightarrow \mathcal{O}(N) \times \left(\mathcal{O}\left(\frac{1}{\sqrt{N}}\right)\right)^3 = \mathcal{O}\left(\frac{1}{\sqrt{N}}\right)$ } Trascurabile

A estensivo $\longrightarrow \frac{1}{N^2}\sqrt{N}^3 \sim \cancel{\mathcal{O}(N)} = \mathcal{O}(\sqrt{N})$

$$W(A) - A\bar{A} = W(A^*) - A^*\bar{A} + \frac{1}{2}\left(\frac{\partial \bar{A}}{\partial A}\right)_{A^*}\underbrace{(A-A^*)^2}_{} + \frac{1}{3!}\left(\frac{\partial^2 \bar{A}}{\partial A^2}\right)_{A^*}(A-A^*)^3 + \dots$$

A intens $\mathcal{O}\left(\frac{1}{\sqrt{N}}\right)$

A estens $\mathcal{O}(\sqrt{N})$

Microcanonico → Canonico

$$Z_{can}(T) = \int \frac{dE}{\Delta} Z_{micro}(E)\, e^{-\beta E}$$

infatti se prendo la mat. dens. discreta non normalizzata

$$\hat{W} = \{ \Delta\, \delta(E - \hat{H}) \}$$

ampiezza della indeterm. energia

$$Z_{can} = \sum_n \frac{\Delta}{\Delta} (\mathrm{tr}\, \delta(E - H))\, e^{-\beta E} = \int_0^\infty \frac{dE}{\Delta} (\mathrm{tr}\, \delta(E - \hat{H}))\, e^{-\beta E}$$

sulle shell di energia

$$= \mathrm{tr}\, e^{-\beta \hat{H}}$$

nel caso classico devo intendere

$$\mathrm{tr}(\ldots) \longrightarrow \int \frac{d^{3N}p\, d^{3N}q}{\Lambda^{3N} N!} (\ldots)$$

in questo caso ho

$$Z_{micro} = e^{S/k}$$ S è il potenziale termodinamico

$$Z_{can}(T) = \int \frac{dE}{\Delta} e^{S/k} e^{-\beta E}$$

dunque la distribuzione di energia è gaussiana intorno a

$$\frac{1}{k} \frac{\partial S(E)}{\partial E} - \beta = 0 \longrightarrow \frac{1}{T} = \left(\frac{\partial S}{\partial E} \right)_{E^*}$$

bagno

Varianza

$$\frac{1}{2k_B}\frac{\partial^2 S}{\partial E^2} = -\frac{1}{2\sigma_E^2}$$

$$-\frac{1}{2\sigma_E^2} = \frac{1}{2k}\frac{\partial}{\partial E}\frac{1}{T} = -\frac{1}{kT^2}\frac{\partial T}{\partial E}$$

$$\sigma_E^2 = kT^2 \frac{1}{\left(\frac{\partial T}{\partial E}\right)_{E^*}} = kT^2\left(\frac{\partial E}{\partial T}\right)_{T^*}$$

Microcan

Canonico

$$\boxed{\sigma_E^2 = kT^2\, C_V}$$

Canonico → Gran canonico

$$Z_{gcan} = \sum_N e^{\beta\mu N} Z(N) =$$

$$= \sum_N e^{\beta[\mu N - F]}$$

$$\frac{\partial}{\partial N}\beta(\mu N - F) = 0 \rightarrow \mu - \left(\frac{\partial F}{\partial N}\right)_{N^*} = 0 \Rightarrow \mu = \mu(N^*)$$

Valor medio di N che ho invertito

fluttuazione

$$\frac{1}{2}\frac{\partial^2}{\partial N^2}[\beta(\mu N - F)] = -\frac{1}{2\sigma_N^2}$$

$$\beta\left(\frac{\partial\mu}{\partial N}\right)_{N^*} = \frac{1}{\sigma_N^2}$$

$$\beta\sigma_N^2 = \underbrace{1\Big/\left(\frac{\partial\mu}{\partial N}\right)_{N^*}}_{\text{Canonico}} = \underbrace{\left(\frac{\partial N}{\partial\mu}\right)_{\mu^*}}_{\text{nel gran canonico}}$$