Random Energy model in Canonical & Microcanonical

ϵ random gaussian zero average

$$Z = \int \frac{d\epsilon}{\sqrt{2\pi\sigma^2}}\, e^{-\beta\epsilon}\, e^{-\frac{\epsilon^2}{2\sigma^2}} = e^{\beta^2\sigma^2/2} > 1$$

energy is not bound from below and above

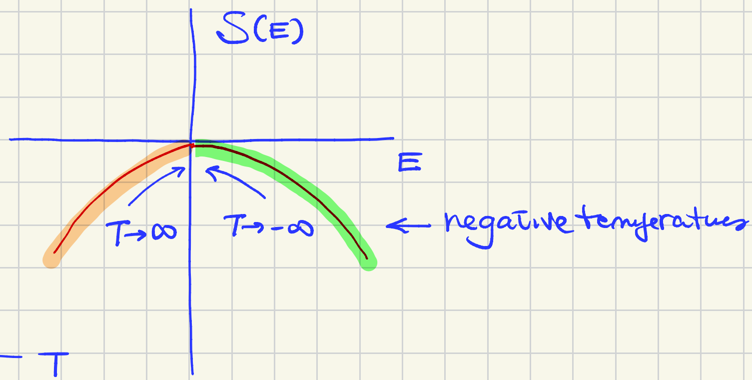
$$U = -\frac{\partial}{\partial\beta}\ln Z = -\beta\sigma^2 < 0 \text{ for any } T>0$$

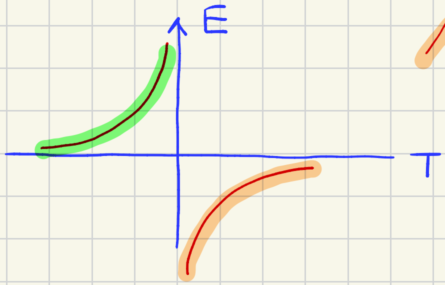
$$\frac{F}{k_B T} = \frac{U}{k_B T} - \frac{\cancel{T}S}{k_B \cancel{T}} \qquad \frac{S}{k_B} = \beta(U-F)$$

$$\frac{S}{k_B} = \beta U + \ln Z = -\beta^2\sigma^2 + \frac{\beta^2\sigma^2}{2} = -\frac{\beta^2\sigma^2}{2} < 0 \text{ for any } T>0$$

Invert $\beta(E) = -\frac{E}{\sigma^2}$

$$\frac{S}{k_B} = -\frac{E^2}{2\sigma^2}$$





Random Energy with box distribution

$$Z = \int_{-\Delta}^{\Delta} \frac{d\epsilon}{2\Delta} e^{-\beta\epsilon} = \frac{1}{\cancel{2}\beta\Delta} \cancel{2}\,ch(\beta\Delta)$$

similar to 2 state model but with $1/\beta$ prefactor

$$U = -\frac{\partial}{\partial\beta} \ln Z = -\Delta\, tgh(\beta\Delta) + \frac{1}{\beta}$$

$$\frac{S}{k_B} = 1 - \beta\Delta\, tgh(\beta\Delta) + \ln\left[\frac{ch(\beta\Delta)}{\beta\Delta}\right]$$

$$\beta E = -\beta\Delta\, tgh(\beta\Delta) + 1$$

$$E = -\Delta\, tgh(\beta\Delta) + \frac{1}{\beta}$$