Matrice densità d'equilibrio e teoria delle perturbazioni MS

$$Z = \mathrm{tr}\, e^{-\beta H} = \mathrm{tr}\, \hat{W}$$

$\hat{W} = e^{-\beta \hat{H}}$ matrice densità d'equilibrio

↳ micro $\Delta \delta(\hat{H} - E)$

↳ can $e^{-\beta \hat{H}}$

↳ grancan $e^{-\beta(\hat{H} - \mu \hat{N})}$

<u>tempo euclideo</u>

$$e^{-\frac{i}{\hbar} t \hat{H}} \longrightarrow e^{-\beta \hat{H}}$$

$\tau = \beta$

$$\boxed{\frac{it}{\hbar} = \tau}$$

<u>tempo immaginario</u>

$$\mathrm{tr}\, e^{-\beta \hat{H}} = \sum_{\{\phi\}} \langle \phi | e^{-\beta \hat{H}} | \phi \rangle$$

$$|\psi, \beta\rangle = e^{-\beta H} |\phi\rangle = U_S(-i\beta\hbar, 0) |\phi\rangle$$

$$U_T(\tau, 0) = U_S(-i\hbar\tau, 0)$$

termica

$$V_I(\tau) = e^{+\tau H_0}\, V\, e^{-\tau H_0}$$

$$U_T(\tau,0) = T_\tau\, e^{-\int_0^\tau d\tau'\, V_I(\tau)}$$

$$\boxed{\mathcal{Z} = \operatorname{tr} e^{-\beta H_0}\, U_T(\beta,0)}$$

$$U_T(\tau,0) = \underbrace{1}_{U^{(0)}} \underbrace{- \int_0^\tau d\tau'\, V_I(\tau')}_{\substack{U^{(1)} \\ \text{con il suo segno}}} \underbrace{+ \frac{1}{2}\int_0^\tau d\tau' \int_0^\tau d\tau''\, T_\tau\, V_I(\tau')\, V_I(\tau'')}_{U^{(2)}} + \dots$$

esattamente

$$\mathcal{Z} = \mathcal{Z}_0 \langle U_T(\beta,0)\rangle_0 = \mathcal{Z}_0\, \frac{\operatorname{tr} e^{-\beta H_0}\, U_T(\beta,0)}{\operatorname{tr} e^{-\beta H_0}}$$

$$\mathcal{Z} = \mathcal{Z}_0 \langle 1 + U_T^{(1)} + \mathcal{O}(V^2)\rangle \quad \text{al } 1^\circ \text{ ordine}$$

La media di un generico operatore

$$\langle \hat{O}\rangle = \frac{\operatorname{tr} e^{-\beta \hat{H}}\, \hat{O}}{\operatorname{tr} e^{-\beta \hat{H}}} \simeq \frac{\left(\operatorname{tr} e^{-\beta H_0}(1 + \hat{U}^{(1)})\,\hat{O}\right)/\mathcal{Z}_0}{\left(\operatorname{tr} e^{-\beta H_0}(1 + \hat{U}^{(1)})\right)/\mathcal{Z}_0}$$

$$\langle \hat{O}\rangle \simeq \frac{\langle (1 + \hat{U}^{(1)})\,\hat{O}\rangle_0}{\langle 1 + \hat{U}^{(1)}\rangle_0} \simeq \langle 1 - U^{(1)}\rangle_0 \langle (1 + U^{(1)})\,\hat{O}\rangle_0$$

$$\langle \hat{O}\rangle \simeq \left[1 - \langle \hat{U}^{(1)}\rangle_0\right]\left[\langle \hat{O}\rangle_0 + \langle U^{(1)}\hat{O}\rangle_0\right]$$

$$\langle \hat{O}\rangle \simeq \langle \hat{O}\rangle_0 + \langle U^{(1)}\hat{O}\rangle_0 - \langle \hat{U}^{(1)}\rangle_0 \langle \hat{O}\rangle_0 + \mathcal{O}(V^2)$$

esparcund

$$\boxed{\langle \hat{O} \rangle - \langle \hat{O} \rangle_0 = - \int_0^\beta d\tau \left\{ \langle \hat{V}_I(\tau) \hat{O} \rangle_0 - \langle V_I(\tau) \rangle_0 \langle \hat{O} \rangle_0 \right\}}$$

se adesso modulo $\hat{V}$ con una sua ampiezza

che mando a zero ottengo

e $\hat{V} = \lambda \hat{v}$ $\quad (H = H_0 + \lambda \hat{v})$

λ ampiezza, $\hat{v}$ operatore di ordine 1

$$\langle \hat{O} \rangle_\lambda - \langle \hat{O} \rangle_0 = -\lambda \int_0^\beta d\tau \left[\langle \hat{v}_I(\tau) \hat{O} \rangle_0 - \langle \hat{v}_I \rangle_0 \langle \hat{O} \rangle_0 \right]$$

dividendo per λ

$$\frac{\langle \hat{O} \rangle_\lambda - \langle \hat{O} \rangle_0}{\lambda} = - \int_0^\beta d\tau \, \underbrace{\left[\langle \hat{v}_I(\tau) \hat{O} \rangle_0 - \langle \hat{v} \rangle_0 \langle \hat{O} \rangle_0 \right]}_{\text{Funzione di correlazione all'equilibrio}}$$

Risposta di $\hat{O}$ a λ perturbazione

$$\frac{\partial}{\partial \lambda} \langle \hat{O} \rangle = R_{0\lambda}$$