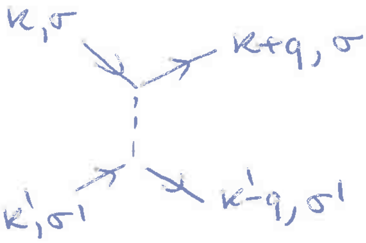
The BCS Model

let's consider a 2 particle interactions for small exchanged momentum we consider a translationally invariant system



σ is the z spin component of electron with momenta ($\hbar = 1$) $\vec{k}$

$c_{\vec{k},\sigma}$ destruction operator for electrons

$$\hat{V} = \frac{1}{2} \sum_{\vec{k}\vec{k}'\vec{q}} \sum_{\sigma\sigma'} V_q \, c^{+}_{k+q,\sigma} \, c^{+}_{k'-q,\sigma'} \, c_{k'\sigma'} \, c_{k\sigma}$$

consider a "Cooper" pair and include the additional constraint

$\sigma = \uparrow \quad \sigma' = \downarrow \qquad \vec{k} + \vec{k}' = 0$ the baricenter of the pair is fixed

then

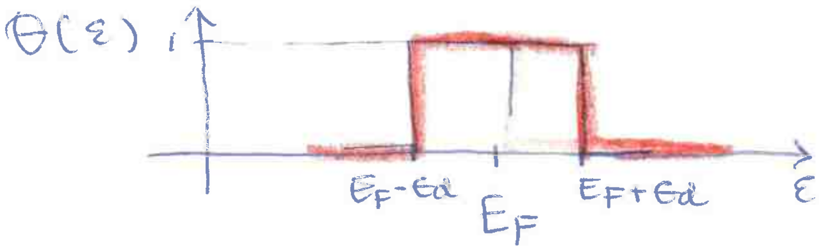
$$\hat{V} = \frac{1}{2} \sum_{\vec{k}\vec{q}} V_{\vec{q}} \, c^{+}_{k+q\uparrow} \, c^{+}_{-k-q,\downarrow} \, c_{-k\downarrow} \, c_{k\uparrow}$$

now $\vec{k}_1 = \vec{k} + \vec{q} \qquad \vec{k}_2 = \vec{k}$

$$\hat{V} = \cancel{\frac{1}{2}} \sum_{\vec{k}_1\vec{k}_2} V(\vec{k}_1 - \vec{k}_2) \, c^{+}_{\vec{k}_1\uparrow} \, c^{+}_{-\vec{k}_1\downarrow} \, c_{-\vec{k}_2\downarrow} \, c_{\vec{k}_2\uparrow}$$

if I consider $\sigma = \downarrow$ and $\sigma' = \uparrow$ I drop 1/2 in front

Now consider the energy shell defined by the function $\Theta_{\vec{k}} = \Theta(\varepsilon_k)$



Replace the translational invariant

$$V(\vec{k}_1-\vec{k}_2) \longrightarrow -V_0\, \Theta_{\vec{k}_1}\Theta_{\vec{k}_2}$$ no longer transl-inv

i.e. a product of 2 functions of $\vec{k}_1$ & $\vec{k}_2$

in such a way that

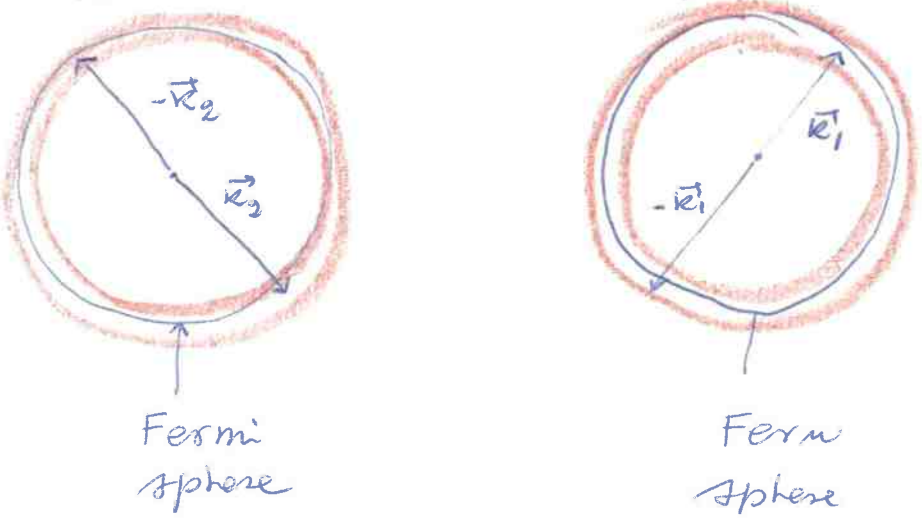
$$\hat{V} = -V_0 \underbrace{\left(\sum_{\vec{k}_1} \Theta_{\vec{k}_1} c^+_{\vec{k}_1\uparrow} c^+_{-\vec{k}_1\downarrow}\right)}_{\hat{A}^+} \underbrace{\left(\sum_{\vec{k}_2} \Theta_2\, c_{-k_2\downarrow}\, c_{k_2\uparrow}\right)}_{\hat{A}}$$

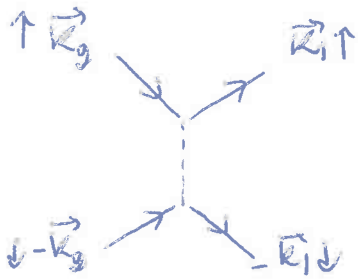
V_0 can be calculated from origine $V(\vec{k}_1-\vec{k}_2)$ as

$$V_0 = -\frac{\int d^3k_1 \int d^3k_2\, \Theta_{k_1}\Theta_{k_2} V(|\vec{k}_1-\vec{k}_2)}{\int d^3k_1 \int d^3k_2\, \Theta_{k_1}\Theta_{k_2}}$$

as an average over energy shells

Interaction represent scattering btw
initial momenta final momenta





interactions takes place only on a resticted shell of amplitude Ed Debye Energy of phonons arond the Fermi surface

Superconducting state is obtained at the Mean Field level for any smale value of (attracting) V_0