Step per BCS

* Modello

$$\hat{K} = \hat{H} - \mu \hat{N} = \sum_{k\sigma} (\varepsilon_k - \mu)\, c^\dagger_{k\sigma} c_{k\sigma} \underbrace{- V_0 \sum_{kk'} \theta_k \theta_{k'}\, c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} c_{-k'\downarrow} c_{k'\uparrow}}_{V}$$

$$A_k = c_{-k\downarrow} c_{k\uparrow} \qquad A^\dagger_k = c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}$$

* linearizzo in $\delta A_k = A_k - \overbrace{\langle A_k \rangle}^{X_k} = A_k - X_k$

$$V = -V_0 \sum_{kk'} \theta_k \theta_{k'} \left(X_k^* + \delta A_k^\dagger \right)\left(X_{k'} + \delta A_{k'} \right) =$$

$$= -V_0 \sum_{kk'} \theta_k \theta_{k'} X_k^* X_{k'} -$$

$$- V_0 \sum_{kk'} \theta_k \theta_{k'} \Big(X_k^* \underbrace{\delta A_{k'}}_{A_{k'} - X_{k'}} + \underbrace{\delta A_k^\dagger}_{A_k^\dagger - X_k^*} X_{k'} \Big)$$

$$+ \cancel{O(\delta A^2)}$$

$$\longrightarrow + 2 V_0 \sum_{kk'} \theta_k \theta_{k'} X_k^* X_{k'}$$

* Ham effettiva $(\hat{K} = \hat{H} - \mu \hat{N})$

$$\hat{K}_{eff} = V_0 \frac{|\Delta|^2}{V_0^2} - \sum_k \left(\Delta^* A_k + \Delta A_k^\dagger \right) + \sum_{k\sigma} (\varepsilon_k - \mu)\, c^\dagger_{k\sigma} c_{k\sigma}$$

$$\boxed{\Delta = \sum_k \theta_k X_k V_0 \quad \Delta^* = \sum_k \theta_k X_k^* V_0} \qquad \Delta = \sum_k \Delta_k$$

$$\hat{K}_{eff} = \frac{|\Delta|^2}{V_0} + \sum_k (\varepsilon_k - \mu) + \underbrace{\sum_k (\varepsilon_k - \mu)\left(c^\dagger_{k\uparrow} c_{k\uparrow} - c_{-k\downarrow} c^\dagger_{-k\downarrow} \right) - \sum_k \Delta^* c_{-k\downarrow} c_{k\uparrow} + \Delta c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}}$$

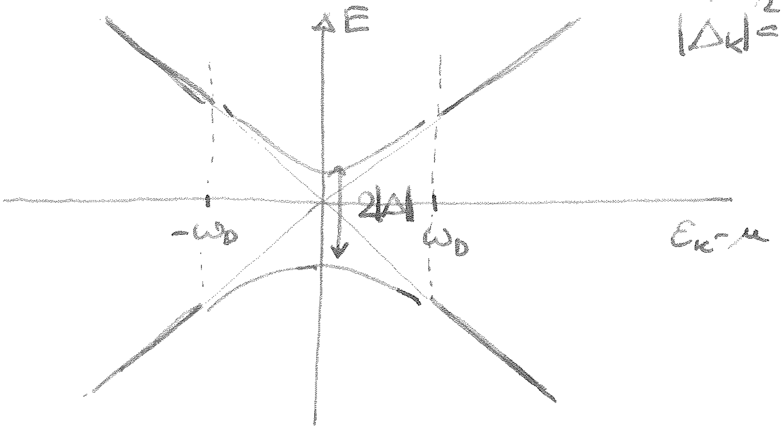
* Diagonalizzo con Bogoliubov

se $\theta_k = 1$

$$\left(c^\dagger_\uparrow \;\; c_\downarrow \right) \begin{bmatrix} \varepsilon - \mu & \Delta \\ \Delta^* & -(\varepsilon - \mu) \end{bmatrix} \begin{pmatrix} c_\uparrow \\ c^\dagger_\downarrow \end{pmatrix}$$

se $\theta_k = 0$ $\quad (c^\dagger_\uparrow \; c_\downarrow)\begin{bmatrix} \varepsilon-\mu & 0 \\ 0 & -(\varepsilon-\mu)\end{bmatrix}\begin{pmatrix} c_\uparrow \\ c^\dagger_\downarrow \end{pmatrix} = (c^\dagger_\uparrow \; c_\downarrow)\begin{pmatrix} (\varepsilon-\mu)c_\uparrow \\ -(\varepsilon-\mu)c^\dagger_\downarrow \end{pmatrix}$

autovalori $\quad E_{k,\alpha} = \pm\sqrt{(\varepsilon_k-\mu) + \underbrace{\theta_k|\Delta|^2}_{|\Delta_k|^2 = \theta_k|\Delta|^2}} = \pm E_k$



$$\hat{K} = \cancel{\sum_{k,\alpha} E_{k,\alpha}\gamma^\dagger_{k,\alpha}\gamma_{k,\alpha}} + \frac{|\Delta|^2}{V_0} + \sum_k(\varepsilon_k-\mu) =$$

$$= \sum_k E_k\gamma^\dagger_{k0}\gamma_{k0} - \gamma_{k1}\gamma^\dagger_{k1}E_k$$

$$\Uparrow$$

$$(\gamma^\dagger_{k0} \; \gamma_{k1})\begin{pmatrix} E_k & 0 \\ 0 & -E_k\end{pmatrix}\begin{pmatrix}\gamma_{k0} \\ \gamma^\dagger_{k1}\end{pmatrix}$$

$$\hat{K} = \sum_k E_k(\gamma^\dagger_{k0}\gamma_{k0} + \gamma^\dagger_{k1}\gamma_{k1}) \underbrace{-\sum_k E_k + \sum_k(\varepsilon_k-\mu)}_{-\sum_k\theta_k E_k + \sum_k\theta_k(\varepsilon_k-\mu)} + \frac{|\Delta|^2}{V_0}$$

*** Traccia su bogoloni** (fermioni a potenziale chimico nullo)

$$\Psi(|\Delta|^2) = \overbrace{\sum_k\theta_k(\varepsilon_k-\mu) - \frac{2}{\beta}\sum_k(1-\theta_k)\ln\left(1+e^{-\beta(\varepsilon_k-\mu)}\right)}^{\Psi_0(T)} +$$

$$+ \frac{2}{\beta}\sum_k\theta_k\ln\left(1+e^{-\beta E_k}\right) - \sum_k\theta_k E_k + \frac{|\Delta|^2}{V_0}$$

$$\left(\Psi = -\frac{1}{\beta}\,\mathrm{tr}\, e^{-\beta\hat{K}_{eff}}\right)$$

* Potenziale di Landau

$$\Psi(|\Delta|) = \Psi_0(T) + \frac{|\Delta|^2}{V_0} \underbrace{- \int_{-\epsilon_d}^{\epsilon_d} d\epsilon\, \mathcal{N}(\varepsilon) E(\varepsilon) - \frac{2}{\beta}\int_{-\epsilon_d}^{\epsilon_d} d\epsilon\, \mathcal{N}(\varepsilon) \ln\left(1+e^{-\beta E(\varepsilon)}\right)}$$

$$E(\epsilon) = \sqrt{\epsilon^2 + |\Delta|^2}$$

$$\downarrow$$

$$E + \frac{2}{\beta}\ln\left(1+e^{-\beta E}\right) = \frac{2}{\beta}\ln 2\mathrm{ch}\left(\beta E/2\right)$$

* Eq della gap

$$\frac{\partial \Psi}{\partial \Delta} = 0$$

$$\Delta = V_0 \Delta \int_0^{\epsilon_d} d\epsilon\, \mathcal{N}(\varepsilon) \frac{\mathrm{tgh}\left(\frac{\beta E(\varepsilon)}{2}\right)}{E(\varepsilon)}$$

$\hookrightarrow$ $\Delta = 0$ stato normale

$\hookrightarrow$ $1 = V_0 \int_0^{\epsilon_d} \cdots$ supercond

* T_c : equazione della gap per $\Delta = 0$

$$1 = V_0 \int_0^{\epsilon_d} d\epsilon\, \mathcal{N}(\varepsilon) \frac{\mathrm{tgh}\, \beta\varepsilon/2}{\epsilon}$$

$\beta \epsilon_d \gg 1$

$$\boxed{k_B T_c = \epsilon_d\, e^{-1/V_0 \mathcal{N}(\epsilon_F)}\ 1{,}13}$$

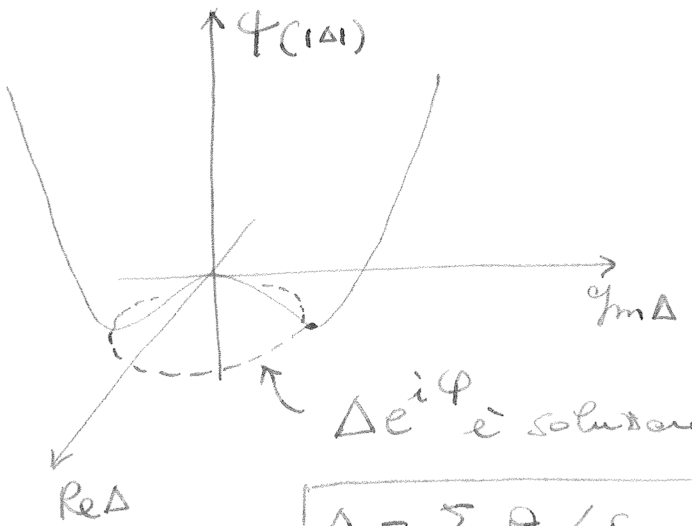
* potenziale di Landau

sviluppo per piccoli $|\Delta|$

$$E - |\epsilon| = \frac{\Delta^2}{2|\varepsilon|} - \frac{\Delta^4}{8|\epsilon|^3} + \theta(\Delta^6)$$

$$\Psi(|\Delta|) \cong \Psi_0(T) + \underbrace{\left[\frac{1}{V_0} - \frac{1}{2}\int_{-\epsilon_d}^{\epsilon_d} d\epsilon\, N(\varepsilon)\, \frac{\mathrm{tgh}\,\beta\varepsilon}{|\varepsilon|} \right]}_{a} |\Delta|^2 +$$

$$+ \underbrace{\left[\int_{-\epsilon_d}^{\epsilon_d} d\varepsilon\, \frac{\mathrm{tgh}\,\beta\varepsilon}{\varepsilon^3} N(\varepsilon) - \frac{\beta}{2}\int d\epsilon\, N(\varepsilon)\, \frac{1}{\varepsilon^2 \mathrm{ch}^2 \frac{\beta\epsilon}{2}} \right]}_{b>0} \frac{|\Delta|^4}{8} + \theta(\Delta^6)$$



$\Delta e^{i\varphi}$ è soluzione con $\forall \varphi$

$$\boxed{\Delta = \sum_k \theta_k \langle c_{-k\downarrow} c_{k\uparrow} \rangle V_0}$$

* Campo "esterno" rottura di gauge

$$\Psi(|\Delta|) = \Psi_0 + a|\Delta|^2 + b|\Delta|^4 - h^*\Delta - h\Delta^*$$

$$H \longrightarrow H \underbrace{- h^* \sum_k \theta_k V_0\, c_{-k\downarrow} c_{k\uparrow} - h \sum_k \theta_k V_0\, c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}}_{\text{NON COMMUTA con } \hat{N} = \sum_{k\sigma} c^\dagger_{k\sigma} c_{k\sigma}}$$