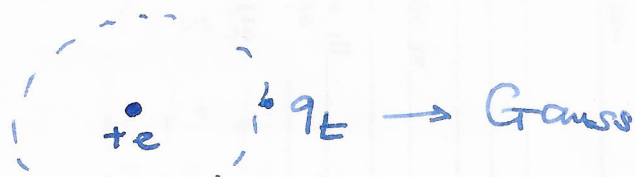


Effective electron-electron interaction & phonon-mediated attraction

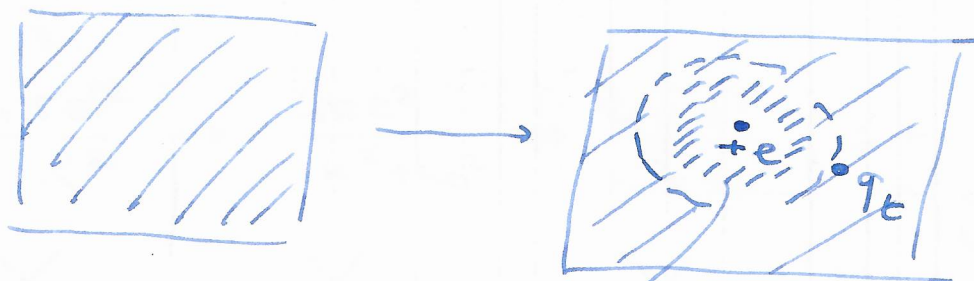
Herman Ashcroft

1) Screening in metals "Solid State Physics"



In vacuum an external fixed charge $+e$ is seen as a bare charge at all distances by a test charge q_t

homogeneous electron gas



accumulation of - charge

a test charge sees an effective charge which is less than $+e$ and depends on distance

$$V(\vec{r}) = \frac{e^2}{r} \rightarrow V(\vec{k}) = \frac{4\pi e^2}{k^2} \quad \text{bare}$$

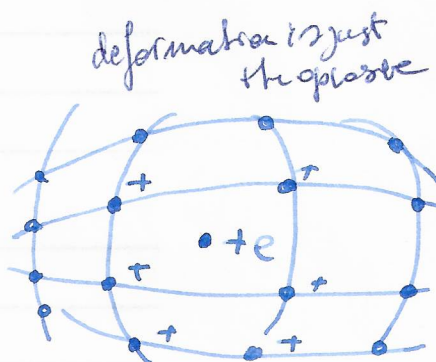
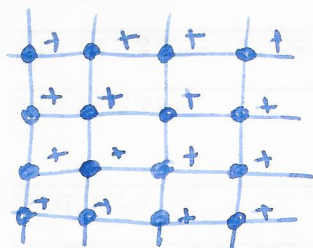
$$V_{\text{screened}}(\vec{k}) = \frac{4\pi e^2}{k^2 + k_0^2} \rightarrow V(\vec{r}) = \frac{4\pi e^2}{r} e^{-r/\xi}$$

$$\xi = \text{screening length} = 1/k_0$$

k_0 = screening wavevector depends on free electron density as free el density $\rightarrow 0$ $k_0 \rightarrow 0$ $\xi \rightarrow \infty$

2) phonons quantized vibration of lattice $\rightarrow$ motion of ions

screening of the positive charged ions



but ions move slowly and screening is not instantaneous but retarded

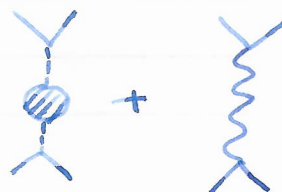
$$\frac{4\pi e^2}{k^2} \rightarrow \frac{4\pi e^2}{k^2 + k_0^2} \rightarrow \frac{4\pi e^2}{k^2 + k_0^2} \left(1 + \underbrace{\frac{\omega(\epsilon)^2}{\omega^2 - \omega(\epsilon)^2}}_{\text{phonon contribution}} \right)$$



bare



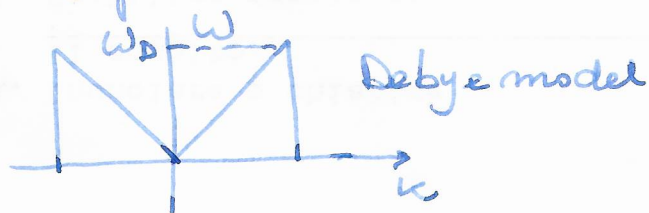
screened



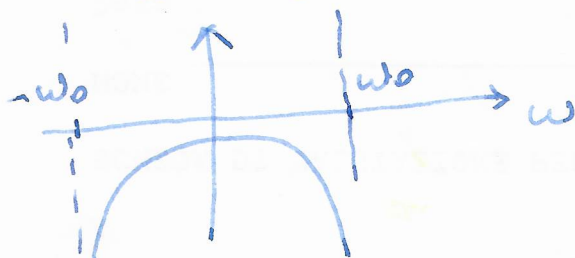
screened + dressed phonon retarded

phonon contribution

$\omega(k)$ phonon dispersion (acoustic $\omega(k) = v_s |k|$)



$$1 + \frac{\omega_0^2}{\omega^2 - \omega_0^2} = \frac{\omega^2}{\omega^2 - \omega_0^2}$$



$$|\omega| < \omega_0$$

phonon contribution is Negative

and can overscreen the repulsive e-e interaction