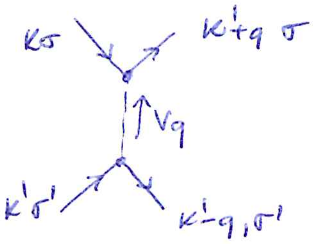
# Decoupling scheme of BCS model

$$\hat{V} = \frac{1}{2} \sum_{\vec{q}\vec{k}\vec{k}' \sigma\sigma'} V_{\vec{q}}\, c^\dagger_{k+q,\sigma}\, c^\dagger_{k'-q,\sigma'}\, c_{k'\sigma'}\, c_{k\sigma}$$

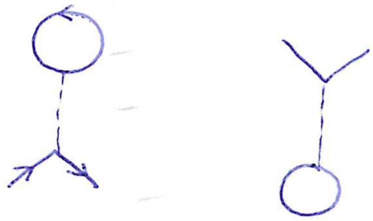



effective chemical potential
propto density

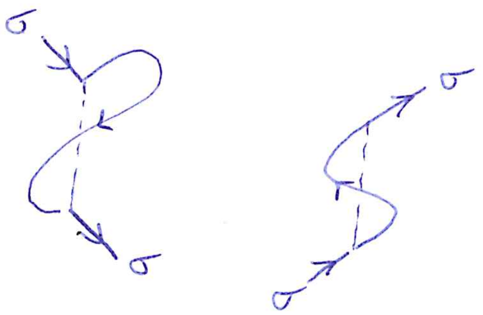
$$\frac{1}{2} \sum_{kk'q \atop \sigma\sigma'} V_q\, c^\dagger_{k+q,\sigma} \langle c^\dagger_{k'-q\,\sigma'} c_{k'\sigma'} \rangle\, c_{k\sigma}$$

$$+\frac{1}{2} \sum_{kk'q \atop \sigma\sigma'} V_q \langle c^\dagger_{k+q,\sigma} c_{k\sigma} \rangle\, c^\dagger_{k'-q\,\sigma'}\, c_{k'\sigma'}$$

Hartree



$$\underbrace{\sum_{k'} \langle c^\dagger_{k'-q,\sigma'} c_{k'\sigma'} \rangle}_{\rho_q}$$

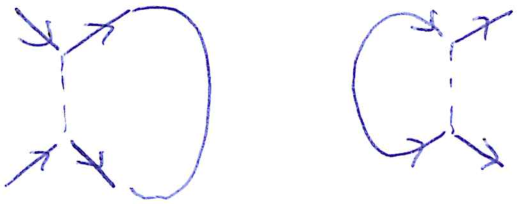



if $V$ indep $\sigma$

Fock

$$-\frac{1}{2} \sum_q V_q \langle c^\dagger_{k+q\,\sigma} c_{k'\sigma'} \rangle\, c^\dagger_{k'-q\,\sigma'}\, c_{k\sigma} \qquad \sigma=\sigma'$$

$$-\frac{1}{2} \sum_q V_q \langle c^\dagger_{k'-q\,\sigma'} c_{k\sigma} \rangle\, c^\dagger_{k+q\,\sigma}\, c_{k\sigma}$$



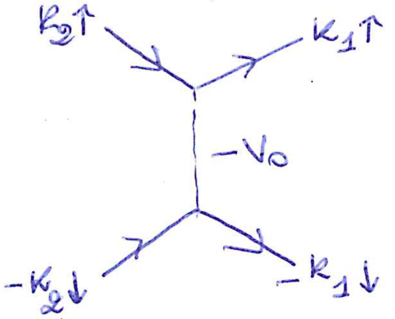
$$\frac{1}{2}\sum_{\substack{kk'\\q\\\sigma\sigma'}} V_q \langle c^\dagger_{k+q,\sigma} c^\dagger_{k'-q,\sigma'}\rangle c_{k'\sigma'} c_{k\sigma}$$

$$\frac{1}{2}\sum_{\substack{kk'\\\sigma\sigma'}} V_q \langle c_{k'\sigma'} c_{k\sigma}\rangle c^\dagger_{k+q,\sigma} c^\dagger_{k'-q,\sigma'}$$

BCS

Breaking the gauge symmetry i.e. not commuting with $\hat{N} = \sum_{k\sigma} c^\dagger_{k\sigma} c_{k\sigma}$

# BCS MODEL




$$= -V_0 \sum_{k_1}\sum_{k_2} \theta_{k_1}\theta_{k_2} c^\dagger_{k_1\uparrow} c^\dagger_{-k_1\downarrow} c_{-k_2\downarrow} c_{k_2\uparrow}$$

we have only 2 k summed and $\sigma = \uparrow$, $\sigma' = \downarrow$

$k+q = k_1 \qquad k' = -k_2$

$k'-q = -k_1 \qquad k = k_2$

the important constraint is that

$$\boxed{k+k'=0}$$

$$\sum_{\substack{kk' \\ q}} \delta_{k,-k'} V_q \, c^\dagger_{k+q\,\sigma} c^\dagger_{k'-q\,\sigma'} c_{k'\sigma'} c_{k\sigma}$$

$$\sum_{kq} V_q \, c^\dagger_{k+q\,\uparrow} c^\dagger_{-k-q\,\downarrow} c_{-k\downarrow} c_{k\uparrow}$$

$$\cancel{\sum_{k_1k_2} V_{k_2-k_1} c^\dagger_{k_2\uparrow} c^\dagger_{-k_2\downarrow} c_{-k\downarrow} c_{k\uparrow}}$$

$\cancel{k+q} = k_2$

$k = k_1$

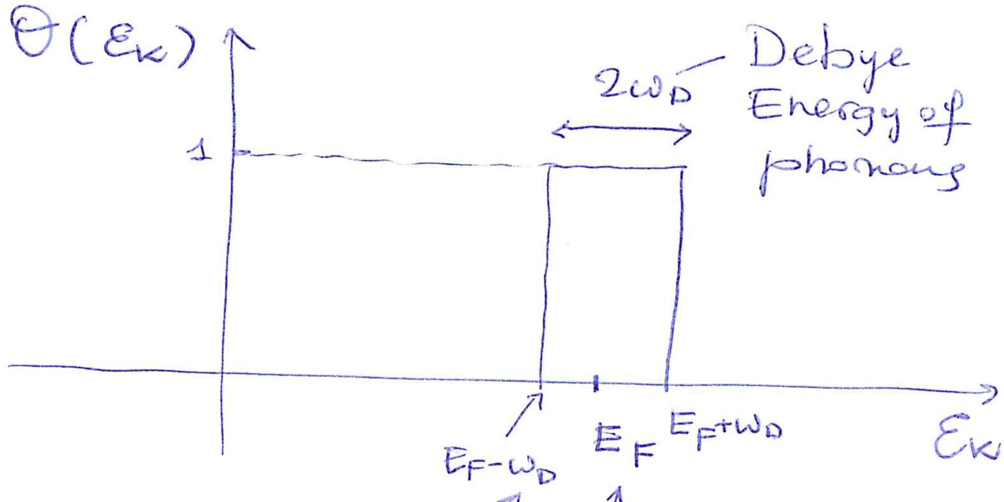
$$\hat{V} = + \sum_{k_1k_2} V_{k_2-k_1} c^\dagger_{k_2\uparrow} c^\dagger_{k_2\downarrow} c_{\mp k\uparrow} c_{\mp k\downarrow}$$

this in case of translational invariant system but BCS is a model which replaces

$$V_{k_2-k_1} \to \theta_{k_1} \theta_{k_2}$$

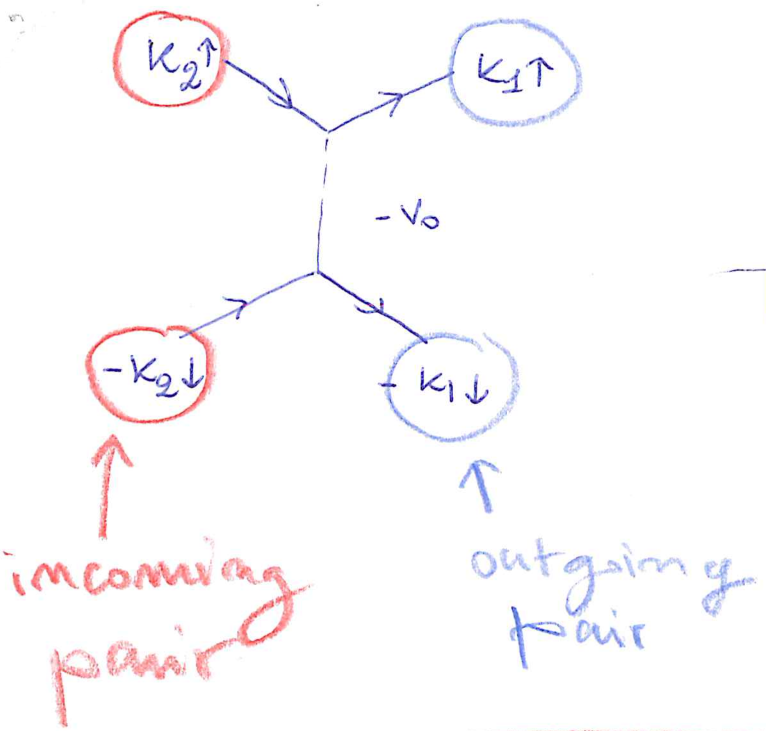
which is solvable in MFT

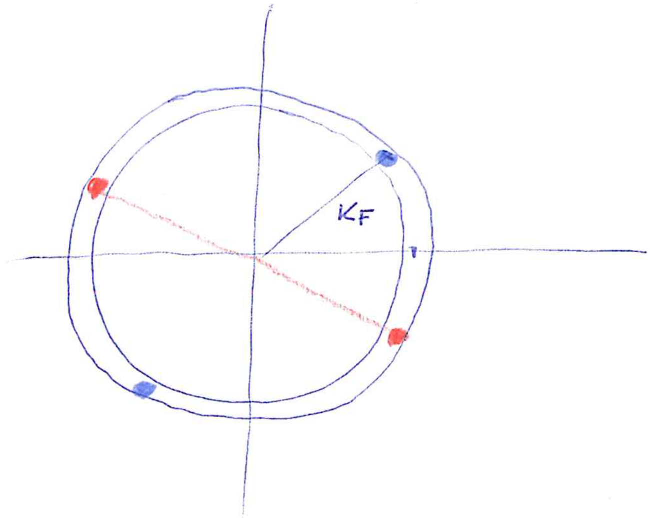
$\theta_k \equiv \theta(\varepsilon_k)$




$\omega_D \sim 10^{-3}$ eV

Fermi energy $\sim$ eV




only particles in a narrow range around Fermi energy are affected by scattering

attractive potential is crucial to destabilise Fermi Liquid (Normal Metal) if a vanishingly small interaction is present at small energies around Fermi energy