Resume of critical exponents

in Landau theory the functional to be now used is expressed as a function of m(x) the order param such as

$$\Psi[m] = \int d^d x \left[a\, m^2(x) + b\,(m^4(x)) + c\,|\vec{\nabla} \cdot \vec{m}|^2 \right]$$

and around T_c $a \propto (T - T_c)$ while $b > 0$ and $c > 0$

this gives rise to the following critical exponent

$$\boxed{m \propto (T_c - T)^\beta \qquad \beta = 1/2}$$

$$\boxed{\chi = \frac{\partial m}{\partial h} \propto (T - T_c)^\gamma \qquad \gamma = 1 \qquad (T > T_c) \quad \text{and } \gamma' = \gamma \ (T < T_c)}$$

$$\boxed{C_h = \left(\frac{\partial U}{\partial T}\right)_h \propto (T_c - T)^\alpha \qquad \alpha = 0}$$

the correlation lenght defined as

$$\boxed{\chi(k) = \frac{\chi(0)}{1 + \xi_e^{-2} k^2} \qquad \xi_e \propto (T - T_c)^{-\nu} \qquad \nu = \frac{1}{2}}$$

Correlation lenght $\to \infty$ $\Rightarrow$ universality and scale relation at $T = T_c$

$$\boxed{m(T=T_c) \propto h^{1/\delta}}$$ $\delta = 3$

critical isotherm

$\eta = 0$

$$\boxed{\chi(q, T=T_c) \propto \frac{1}{q^{2-\eta}}}$$

All theese exponents being dimensionality dependent and called CLASSICAL EXPONENTS.

In real systems however there are important dependence on

dimensionality of space

# of components / symmetry of order parameter

For scalar order parameter such as Ising Model or gas liquid transition Lattice Gas

Besides the scaling (which holds at the critical point) the field theory is Conformal (at $T = T_c$) and all the critical exponents can be derived from the scaling dimensions of

$$m(x) \xrightarrow{\Lambda} \Lambda^{\sigma} m$$ previously called x_m

$$m^2(x) \xrightarrow{\Lambda} \Lambda^{\varepsilon} m^2$$ x_{m^2}

$$m^4(x) \xrightarrow{\Lambda} \Lambda^{\varepsilon'} m^2$$ x_{m^4}

it is fend that the classical critical exponent are that found for $d=4$ for example

	$d=1$	$d=2$	$d=3$	$d=4$
β	no transition	$1/8$	$0{,}326419(3)$	$1/2$
ν	no trans	1	$0{,}6299711(4)$	$1/2$

Scaling relations relates all critical exponent but two for example

$$x_h + x_m = d$$

scaling dimension of m

scaling dimension of h

and the scaling laws involves dimensionality such that they are fullfilled by the classical critical exponent only at $d=4$

therefore Mean field theory is

- Quantitatively not correct below an Upper critical dimension $d_u = 4$ (ISING)
- Qualitatively incorrect below the lower critical dimension

$$d_\ell = 2 \text{ (ISING)}$$

(no phase trans or changing of type of pt)