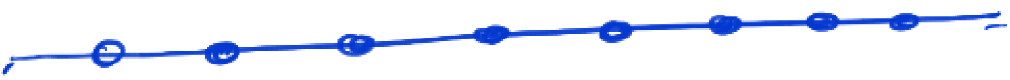
Matrici di trasferimento

Modello di Ising a d=1

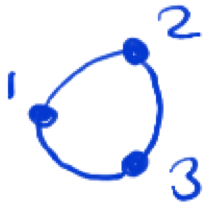
$$H = -\frac{J}{2}\underbrace{\sum_{\langle ij\rangle}}_{\text{primi vicini}}\sigma_i\sigma_j - h\sum_i \sigma_i$$

$\sigma_i = \pm 1$

$d=1$



+ condizioni periodiche



$$-\frac{J}{2}\sum_{\langle ij\rangle}\sigma_i\sigma_j = -\frac{J}{2}\big(\underbrace{\sigma_1\sigma_2+\sigma_1\sigma_3}_{i=1}$$
$$\underbrace{\sigma_2\sigma_3+\sigma_2\sigma_1}_{i=2}$$
$$\underbrace{\sigma_3\sigma_1+\sigma_3\sigma_2}_{i=3}\big)$$

$$= -J(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)$$

$$H = -J\sum_{i=1}^{N}\sigma_i\sigma_{i+1} - h\sum\sigma_i \quad + \text{PBC}$$

per $i = N$ $\quad \sigma_N \sigma_1$

$$e^{-\beta H} = e^{\beta J \sum_i \sigma_i \sigma_{i+1}} \, e^{\beta h \sum_i \sigma_i}$$

$$\boxed{T(\sigma, \sigma') = e^{\beta J \sigma \sigma' + \beta h \sigma}}$$

$N = 3$

$$e^{-\beta H} = e^{\beta J (\sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1) + \beta h (\sigma_1 + \sigma_2 + \sigma_3)}$$

$$= T(\sigma_1 \sigma_2) \, T(\sigma_2 \sigma_3) \, T(\sigma_3 \sigma_1)$$

$$\mathrm{tr}\, e^{-\beta H} = \sum_{\sigma_1 = -1}^{1} \sum_{\sigma_2 = -1}^{1} \sum_{\sigma_3 = -1}^{1} T(\sigma_1 \sigma_2) \, T(\sigma_2 \sigma_3) \, T(\sigma_3 \sigma_1)$$

$$= \mathrm{tr}\, \mathbb{T}^3$$

$T(\sigma, \sigma')$ è una matrice 2×2

$$\mathbb{T} = \begin{array}{cc} & \begin{array}{cc} + & - \end{array} \\ & \begin{pmatrix} e^{\beta(J+h)} & e^{\beta(h-J)} \\ e^{-\beta(J+h)} & e^{\beta(J-h)} \end{pmatrix} \begin{array}{c} + \\ \end{array} \end{array}$$

$$\left| e^{\dots} \quad e^{\beta \dots} \right| -$$

$$Z = \mathrm{tr}\, \mathbb{T}^N = \lambda_1^N + \lambda_2^N$$

dove λ_1 e λ_2 sono gli autovalori di $\mathbb{T}$

Questo da la soluzione esatta del modello di Ising $d=1$

$$\lambda_{12} = e^{\beta J} ch(\beta h) \pm \sqrt{e^{2\beta J} ch^2(\beta h) - 2 sh(2\beta J)}$$

per $h=0$

$$\lambda_{12} = (e^{\beta J} \pm e^{-\beta J})$$

$$m = \langle \sigma_i \rangle = \frac{\mathrm{tr}\, e^{-\beta H} \sigma_i}{\mathrm{tr}\, e^{-\beta H}}$$

$$m = \frac{\mathrm{tr}\, \overbrace{\mathbb{T}\,\mathbb{T}\,\mathbb{T}}\, \overset{i=3}{\sigma}\, \overbrace{\mathbb{T} \cdots \mathbb{T}}}{}$$

$$\mathrm{tr}\, T^N$$

$$= \frac{\mathrm{tr}\, T^N \sigma}{\mathrm{tr}\, T^N}$$

$$m = -\frac{1}{N}\frac{\partial F}{\partial h} \qquad F = -\frac{1}{\beta}\ln Z$$

$$m = \frac{\mathrm{sh}(\beta h)}{\sqrt{\mathrm{sh}^2(\beta h) + e^{-4\beta J}}}$$

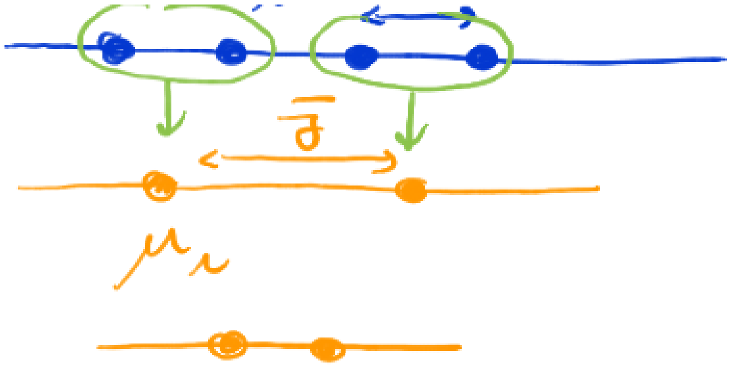
per piccolo h

$$m \simeq \frac{\beta h}{\sqrt{e^{-4\beta J}}} \sim e^{2\beta J}\beta h \underset{(h\to 0)}{\to} 0$$

per $\forall\, T > 0$ $\quad \lim_{h\to 0} m(h,T) = 0$

Sfruttando queste possibilità di soluzione

$\sigma_i \qquad J$



μ_i