d	n	
1	question	no
2	$n=1$	si
2	$n=2$	Kosterlitz-Touless
2	$n>2$	no
3	$n=1$	si
3	$n>2$	si

$n \geq 2$ continuous symmetry

$\Rightarrow$ no symmetry breaking in $d \leq 2$

$n=2$ $d=2$ Kosterlitz Touless

TEOREMA Mermin & Wagner

Mermin-Wagner

- per simmetrie continue $n \geq 2$
- per interazioni a <u>corto raggio</u>
- per dimensioni spaziali $d \leq 2$
- a temperature > 0

Non esistono rotture spontanee di simmetria

$d=2$ $n=2$ Kosterlitz-Touless

$m = 0$ ma $\xi \to \infty$

TEOREMA Mermin & Wagner

d	n	
1	qualsiasi	no
2	$n=1$	si
2	$n=2$	Kosterlitz-Touless
2	$n>2$	no
3	$n=1$	si
3	$n>2$	si

$n \geq 2$ continuous symmetry

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Non esistono rotture spontanee di simmetria

$d=2$ $n=2$ Kosterlitz-Touless

$m=0$ ma $\xi \to \infty$

$$\mathcal{Z} = \mathrm{tr}\, e^{-\frac{\beta}{V}\sum_k (ck^2+a)\vec{\varphi}_k\cdot\vec{\varphi}_{-k} + \frac{\beta}{V}\sum_k \vec{h}_{-k}\cdot\vec{\varphi}_k}$$

supponiamo

$\langle\vec{\varphi}_k\rangle = m_\parallel \hat{u}_\parallel$ mentre nella parte $\perp$ $\langle\vec{\varphi}_\perp\rangle = 0$

$$\vec{\varphi}_k\cdot\vec{\varphi}_{-k} = m_\parallel^2 + (\vec{\varphi}_k - m_\parallel\hat{u}_\parallel)\cdot(\vec{\varphi}_{-k} - m_\parallel\hat{u}_k)$$

$$\vec{\varphi}_k = \vec{\varphi}_\parallel + \vec{\varphi}_\perp \qquad \varphi_\parallel = m_\parallel + \delta\varphi_\parallel$$

$$\vec{\varphi}_k\cdot\vec{\varphi}_{-k} = \vec{\varphi}_{\parallel k}\cdot\vec{\varphi}_{\parallel -k} + \vec{\varphi}_{\perp k}\cdot\vec{\varphi}_{\perp -k} = m_\parallel^2 + \delta\varphi_{\parallel k} m_\parallel + m_\parallel\delta\varphi_{\parallel -k} + \delta\varphi_k\cdot\delta\varphi_{-k} + \vec{\varphi}_\perp\cdot\vec{\varphi}_\perp$$

$$\mathcal{Z} = \left[\prod_k \left(\frac{\pi V}{(ck^2+a)\beta}\right)^{\cancel{1/2}}\right] e^{\frac{\beta}{4V}\sum_k \frac{|h_k|^2}{ck^2+a}}$$

$$F = \frac{1}{\cancel{2}\beta}\sum_k \ln\left(\frac{ck^2+a}{\pi k_B T}\right) - \frac{1}{4V}\sum_k \frac{|h_k|^2}{ck^2+a} \qquad T<T_c$$

Mermin & Wagner

$\frac{b}{V^3}\varphi_k\varphi_{k'}\varphi_{-k'-q}\varphi_{-k+q}$ sviluppo gaussiano

$$\varphi_k = V\delta_k m + \delta\varphi_k$$

$$\frac{b}{V^3}\sum_{kk'q}(V\delta_k m + \delta\varphi_k)(V\delta_{k'} m + \delta\varphi_{k'})(V\delta_{-k'-q} m + \delta\varphi_{-k'-q})(V\delta_{-k+q} m + \delta\varphi_{-k+q})$$

bVm^4

$4b\,\delta\varphi_0 m^3$ — $\varphi_0 = \int d^dx\,\varphi(x) \quad \delta\varphi_0 = \int d^dx\,\delta\varphi(x) = 0$

$6\frac{b}{V}\sum_k \delta\varphi_k\delta\varphi_{-k} m^2 \quad + \mathcal{O}(\delta\varphi^3)$

rinorm. termine $\square$ H_0

$T < T_c \qquad a \to a + 6bm^2 \qquad$ con $a < 0$

ma $a + 6bm^2 > 0$ perché

$2am + 4bm^3 = 0 \qquad \cancel{2a} \quad a = -2bm^2$ per $T < T_c$

$T > T_c$

$$\frac{1}{V}\langle \varphi_k \varphi_{-k} \rangle = \frac{1}{2\beta c} \frac{1}{k^2 + \xi^{-2}} \qquad \xi_1^{-2} = a/c$$

$T < T_c$

$$\frac{1}{V}\langle \varphi_k \varphi_{-k} \rangle = \frac{1}{2\beta c} \frac{1}{k^2 + \xi^{-2}} \qquad \xi_1^{-2} = \frac{a + 6bm^2}{c}$$

Parametro d'ordine multi comp

$$\int d^d x \left[|\vec{\varphi}(x)|^2 \right]^2 = \int d^d x \left(\varphi_\parallel^2 + \vec{\varphi}_\perp^2 \right)^2 =$$

$$= \int d^d x \left[\varphi_\parallel^4 + \vec{\varphi}_\perp^4 + 2\varphi_\parallel^2 \varphi_\perp^2 \right]$$

fluttuazioni $\varphi_\parallel = m + \delta\varphi \qquad \varphi_\perp$ tutta fluttuazione

$$\int d^d x \left[m^4 + \cancel{4 m \delta\varphi_\parallel} + 6m^2 \delta\varphi_\parallel^2 + \mathcal{O}(\delta\varphi^3) + 2\left(m^2 + \delta\varphi_\parallel^2 + \cancel{2m\delta\varphi_\parallel}\right)\varphi_\perp^2 \right]$$

$$= \int d^d x \left[6m^2 \delta\varphi_\parallel^2 + 2m^2 \varphi_\perp^2 \right] + \mathcal{O}(\delta\varphi^3) + \text{const}$$

multi componente

$$T < T_c \longrightarrow \quad a_\parallel \to a_\parallel + 6bm^2$$

$$a_\perp \longrightarrow a_\perp + 2bm^2$$

ma allora per $T < T_c$

$a_\perp + 2bm^2 = 0$ sempre!

equazione di Curie Weiss $(h \neq 0)$

$$2am + 4bm^3 - h = 0$$

$$(a + 2bm^2)m = h/2 \qquad a + 2bm^2 = h/2m$$

$$C_{\parallel} = (\text{const}) \frac{1}{q^2 + \underbrace{\frac{a+6bm^2}{c}}_{\xi_{\parallel}^{2}}} = (\text{const}) \frac{1}{q^2 + \frac{4bm^2}{c} + \frac{h}{2m}}$$

$$C_{\perp} = (\text{const}) \frac{1}{q^2 + \underbrace{\frac{a+2bm^2}{c}}_{\xi_{\perp}^{2}}} = (\text{const}) \frac{1}{q^2 + h/2m}$$

$$C_{\perp}(r) = (\text{const}) \int d^d q \frac{e^{i\vec{q}\cdot\vec{r}}}{q^2 + \underbrace{\left(\frac{h}{2m}\right)}_{\xi_{\perp}^{-2}}} = (\text{const})\, \xi_{\perp}^{2-d} \int d^d x \frac{e^{i\vec{x}(\vec{r}/\xi_{\perp})}}{x^2 + 1}$$

$d \leq 2 \quad \xi_{\perp} \to \infty$

$$C_{\perp} = (\text{const})\, \xi_{\perp}^{2-d} \int d\Omega^{d-1} \int_0^\infty dx\, x^{d-1} \frac{e^{ix\frac{r}{\xi}\cos\theta}}{1+x^2} \qquad n \geq 2$$

$n = 2 \quad d = 2$
Marginale

se $d-3 < -1 \quad d < 2$ converge
e la divergenza è espressa da questo

$C_{\perp}$ divergerebbe a $\forall\, T < T_c \Rightarrow$ l'ordine è rotto
dalle fluttuazioni $\perp$

nello spazio reale !