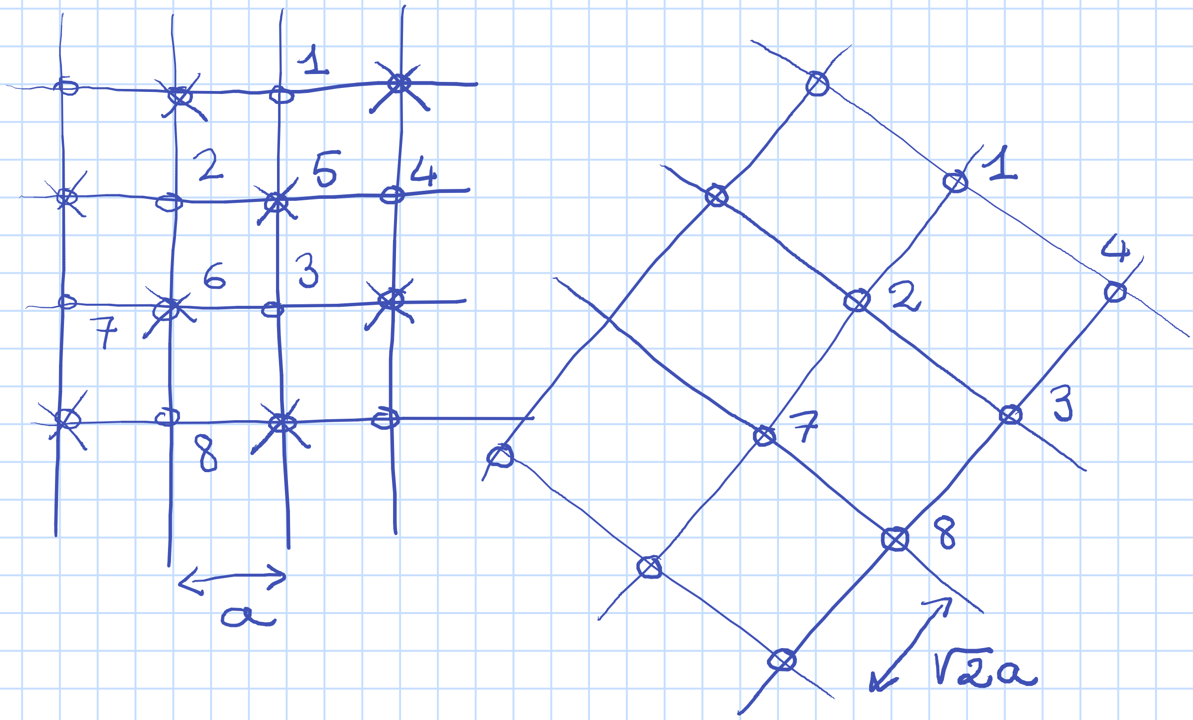
RG flow in 2-d Ising Model from Chandler 5.7 + calculation of index α



Decimation proceeds by integrating out (tracing the ⊗ spins)

here the situation is much more complex than in $d=1$

The idea is to write

$$Z = \mathrm{tr}\, e^{-\beta H} = \mathrm{tr}_{\circ}\, e^{-\beta[\bar{H}+\tilde{F}]}$$

where $e^{-\beta \bar{H}} e^{-\beta \tilde{F}} = \mathrm{tr}_{\otimes}\, e^{-\beta H}$

A

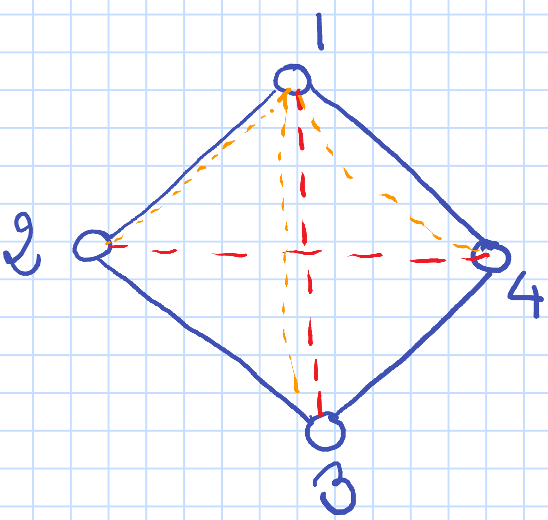
this is formal the scaling hypothesis
is that if

$$\mathcal{H} = -J \sum_{i<j}^{(all)} \sigma_i \sigma_j \qquad (h=0)$$

then

$$\bar{\mathcal{H}} = -\bar{J} \sum_{i<j}^{(only\ 0)} \sigma_i \sigma_j$$

Unfortunately this is not possible in the d=2 Ising model
We need further parameters



— J_1 } 2 spin interaction
— J_2
— J_4 ← 4 spin interaction

therefore

$$\bar{\mathcal{H}} = -\bar{J}_1 \underbrace{\sum_{\langle ij \rangle}^{(0)}}_{\text{nearest neighbor}} \sigma_i \sigma_j - \bar{J}_2 \underbrace{\sum_{\langle\langle ij \rangle\rangle}^{(0)}}_{\text{next nearest}} \sigma_i \sigma_j - \bar{J}_4 \sum_{ijkl}^{(0)} \sigma_i \sigma_j \sigma_k \sigma_l$$

the solution is shown on the book of Chandler we here report the results

if $k = \beta J$

$$\bar{k}_1 = \frac{1}{4} \ln(\mathrm{ch}(4k))$$

$$\bar{k}_2 = \frac{1}{8} \ln(\mathrm{ch}(4k))$$

$$\bar{k}_4 = \frac{1}{8} \ln[\mathrm{ch}(4k)] - \frac{1}{2} \ln[\mathrm{ch}(2k)]$$

$$A = 2(\mathrm{ch}(2k))^{1/2}(\mathrm{ch}(4k))^{1/8}$$

$$e^{-\beta \tilde{F}} = A^{N/2}$$

if we neglect k_2 and k_4 we arise to an equation for $\bar{k}$ similar to $d=1$ Ising model i.e. we have only TRIVIAL fixed points

We need to take into account at least k_2

to compute an effective K_{eff} which takes into account both nearest-neighbor and next nearest neighbor in an approximate manner

$$\mathcal{H} = \overbrace{-J_1 \sum_{\langle ij \rangle} \sigma_i \sigma_j}^{H_1} \overbrace{- J_2 \sum_{\langle\langle ij \rangle\rangle} \sigma_i \sigma_j}^{H_2}$$

$$\downarrow$$

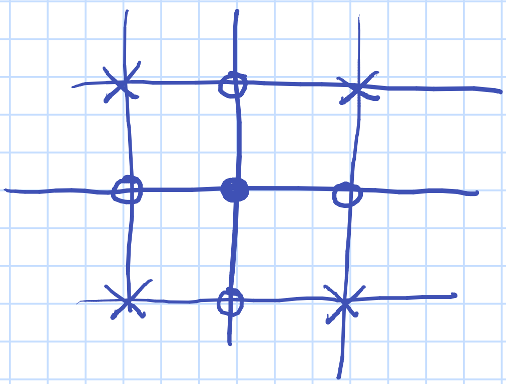
$$-J_{eff} \sum_{\langle ij \rangle} \sigma_i \sigma_j$$

let's consider the state in which all σs are $\uparrow$

$$H_1 = -J_1 N z$$

$$H_2 = -J_2 N z_2$$

z_2 = # of next nearest neighbor



$\circ$ = nn

$\times$ = nnn

$$z_2 = z_1$$

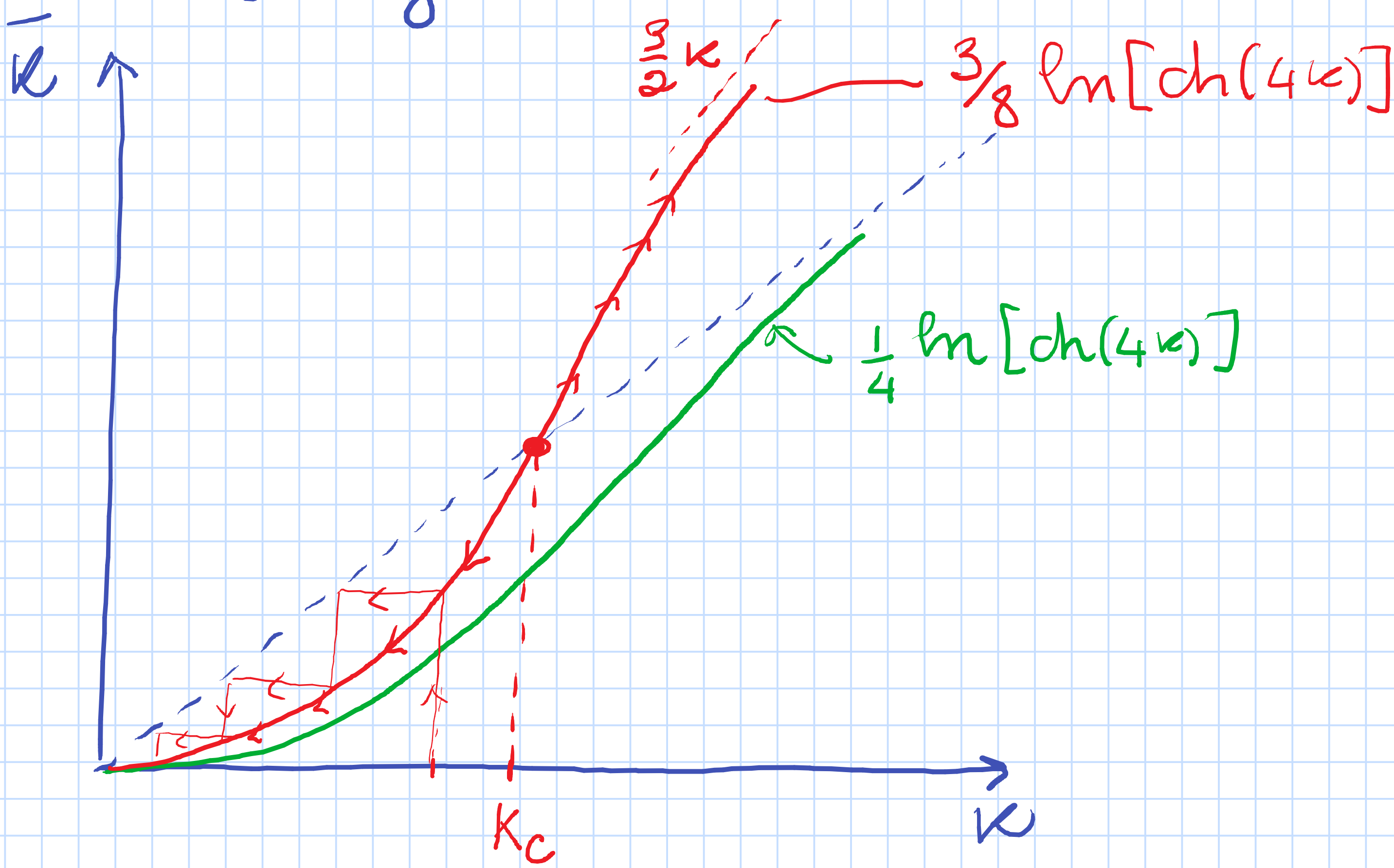
$$\simeq \quad J_{eff} \simeq J_1 + J_2$$

we arrive to the following approximate flow equation

$$\bar{K} = \frac{3}{8} \ln ch(4K)$$

despite the modest change of prefactor from $1/4 \to 3/8$ this equation admit a non trivial fixed point

$$K_c = \frac{3}{8} \ln [ch(4K_c)]$$



this fixed point

$$K_c \simeq 0{,}507$$

is unstable i.e. if one starts

with $K_0 < K_c$ then $\bar{K} \to 0$ } Trivial

$K_0 > K_c$ then $\bar{K} \to \infty$ } fixed points

this has a physical interpretation

if $T > T_c$ $K_0 < K_c$

and $\bar{K} \to 0$ means that at large scales one see the system as fully disordered

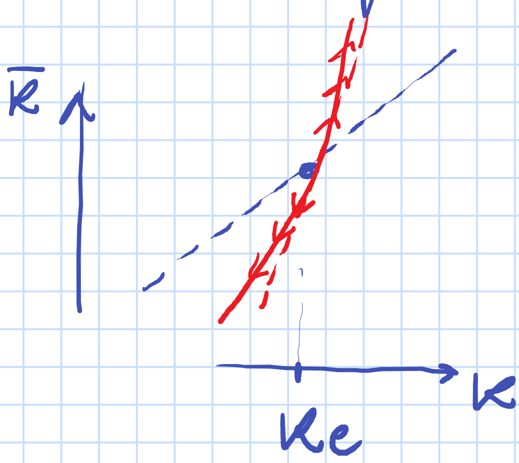
if $T < T_c$ $K_0 > K_c$

and at large scales the system is fully ordered

Let us see how the explicit form of RG recursion allows the calculation of the critical index —

Calculation critical indexes 1nd=2 Ising

linearizing the RG flow around the fixed point



$$\bar{K} = g(K)$$

$$\bar{K} = K_c + \left.\frac{dg}{dK}\right|_{K_c} (K - K_c) + O(K-K_c)^2$$

$$\frac{\bar{K} - K_c}{K - K_c} \simeq \left(\frac{dg}{dK}\right)_{K_c}$$

being $K = \beta J$

$$\frac{\bar{K} - K_c}{K - K_c} = \frac{\bar{\beta} - \beta_c}{\beta - \beta_c} = \frac{T_c - \bar{T}}{T_c - T} \frac{T T_c}{\bar{T} T_c} \simeq \frac{\bar{t}}{t}$$

~ 1

$$\frac{\bar{t}}{t} = L^{x_t} = \left(\frac{dg}{\partial k}\right)_{kc}$$

$$x_t = \frac{1}{\ln L} \ln\left(\left(\frac{dg}{dk}\right)_{kc}\right)$$

but from page 1 $L = \sqrt{2}$

$$\left(\frac{dg}{dk}\right)_{kc} = \frac{3}{8} 4 \, tgh(4 \cdot 0{,}507) \simeq 1{,}5$$

$$\nu = \frac{1}{x_t} = 0{,}934$$

and using the scaling relation

$$\alpha = 2 - d\nu \simeq 0{,}131$$

we have performed h=0 calculation therefore only these two indexes can be calculated. We need x_h by an $h \neq 0$ calculation