Dalla teoria delle pert su

$H = H_0 + V$ (ingloba λ i.e. 1) — CALCOLO AL 1° ORDINE DELLA F. di CORR

ho:

$$\langle \hat{O} \rangle^{(1)} = \langle \hat{O} \rangle_0 - \left[\langle \int_0^\beta d\tau \, \hat{V}(\tau) \hat{O} \rangle_0 - \langle \int_0^\beta d\tau \hat{V}(\tau) \rangle_0 \langle \hat{O} \rangle \right]$$

ora se faccio la "perturbazione" intorno ad una ham effettiva

$$\hat{V} = (H - H_{eff})$$

e calcolo per $\hat{O}$ una funzione di correlazione spaziale ad ugual tempo ho

$$\langle \hat{O} \rangle^{(1)} = \langle \hat{O} \rangle_{eff} - \left[\langle \int_0^\beta d\tau \, (H - H_{eff})(\tau) \hat{O} \rangle_{eff} - \int_0^\beta d\tau \langle H - H_{eff}(\tau) \rangle_{eff} \langle \hat{O} \rangle_{eff} \right]$$

$$[H_{eff}, e^{\tau H_{eff}}] = 0 \Rightarrow H_{eff} \text{ indip } \tau$$

$$\langle H(\tau) \rangle_{eff} = \frac{1}{Z_{eff}} \operatorname{tr} e^{-\beta H_{eff}} e^{\tau H_{eff}} H e^{-\tau H_{eff}} = \langle H \rangle_{eff}$$

indip τ

$$\langle \hat{O} \rangle^{(1)} = \langle \hat{O} \rangle_{eff} - \left[\int_0^\beta d\tau \langle H(\tau) O \rangle_{eff} - \beta \langle H_{eff} \hat{O} \rangle - \beta \langle H \rangle_{eff} \langle \hat{O} \rangle_{eff} + \beta \langle H_{eff} \rangle \langle \hat{O} \rangle_{eff} \right]$$

l'unico termine che dip da τ

$$= \langle \hat{O} \rangle_{eff} - \left[\int_0^\beta d\tau \, \langle H(\tau) O \rangle^c_{eff} - \beta \langle H_{eff} \hat{O} \rangle^c_{eff} \right]$$

Nel caso del modello di Ising

$$\hat{O} = \sigma_i \sigma_j$$

$$\langle \sigma_i \sigma_j \rangle^{(1)} = \langle \sigma_i \sigma_j \rangle_{eff} - \beta \left[\langle H \sigma_i \sigma_j \rangle^c_{eff} - \langle H_{eff} \sigma_i \sigma_j \rangle^c_{eff} \right]$$

$$\langle \sigma_i \sigma_j \rangle_{eff} = (1 - m_i^2)\, \delta_{ij}$$

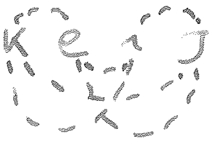
$$\langle H \sigma_i \sigma_j \rangle^c_{eff} = \underbrace{\langle H \sigma_i \sigma_j \rangle_{eff}}_{} - \langle H \rangle_{eff} \langle \sigma_i \sigma_j \rangle_{eff}$$

parte + interessante

$$\langle \left(-J \textstyle\sum_{kl} \sigma_k \sigma_l - h \sum_k \sigma_k \right) \sigma_i \sigma_j \rangle_{eff}$$

$$\langle \sigma_k \sigma_l \sigma_i \sigma_j \rangle_{eff} = \cancel{\langle \sigma_k \sigma_l \rangle \langle \sigma_i} \sigma_j \rangle$$

$$\langle \sigma_k \sigma_i \rangle \langle \sigma_l \sigma_j \rangle$$



$$\langle \sigma_k \sigma_j \rangle \langle \sigma_l \sigma_i \rangle$$

$k \neq l$

$$\langle \sigma_k \rangle \langle \sigma_l \rangle \langle \sigma_i \sigma_j \rangle$$

meglio passare alle variabili sottratte

$u_i = \sigma_i - m_i$ per modo che $\langle u_i \rangle = 0$ $\langle u_i u_j \rangle = \delta_{ij}(1 - m_i^2)$

$$\langle u_k u_l u_i u_j \rangle = \cancel{\delta_{kl}(1 - m_l^2)\, \delta_{ij}}\, (1 - m_i^2) \quad \text{perché } k \neq l$$

$$+ \delta_{ki}(1 - m_i^2)\, \delta_{lj}(1 - m_j^2)$$

$$+ \delta_{kj}(1 - m_j^2)\, \delta_{li}(1 - m_i^2)$$

$$u_i^2 = \sigma_i^2 + m_i^2 - 2\sigma_i m_i = 1 + m_i^2 - 2\sigma_i m_i$$

$$u_i^3 = \sigma_i^3 + m_i^3 - 3\sigma_i^2 m_i + 3 m_i^2 \sigma_i \quad \langle u_i^3 \rangle = m_i + m_i^3 - 3m_i + 3m_i^3$$

$$\langle u_k u_\ell u_\lambda u_j\rangle = \delta_{k\lambda}(1-m_\lambda^2)\,\delta_{\ell j}(1-m_j^2) +$$

$$+\,\delta_{kj}(1-m_j^2)\,\delta_{\ell\lambda}(1-m_\lambda^2)$$

$$\sum_{\langle k\ell\rangle}\langle\sigma_k\sigma_\ell\sigma_\lambda\sigma_j\rangle = \sum_{\langle k\ell\rangle}\langle(\underbrace{\sigma_k-m_k}_{u_k}+m_k)(\underbrace{\sigma_\ell-m_\ell}_{u_\ell}+m_\ell)(\underbrace{\sigma_\lambda-m_\lambda}_{u_\lambda}+m_\lambda)(\underbrace{\sigma_j-m_j}_{u_j}+m_j)\rangle$$

$$= \sum_{\langle k\ell\rangle} m_k m_\ell m_\lambda m_j +$$

$$+ \sum_{\langle k\ell\rangle} m_k \cancel{m_\ell m_\lambda\langle u_\lambda\rangle} + m_k \cancel{m_\ell\langle u_\lambda\rangle} m_j + \cdots$$

$$+ \sum_{\langle k\ell\rangle} m_k m_\ell\,\delta_{\lambda j}(1-m_\lambda^2) + m_k m_\lambda\,\delta_{\ell j}(1-m_j^2) + m_k m_j\,\delta_{\ell\lambda}(1-m_\lambda^2) +$$

$$+ \sum_{\langle k\ell\rangle} \cancel{\delta_{k\ell}(1-m_\ell^2)}\, m_\lambda m_j + \delta_{k\lambda}(1-m_\lambda^2)\, m_\ell m_j + \delta_{kj}(1-m_j^2)\, m_\ell m_\lambda$$

$$+ \sum_{\langle k\ell\rangle} \delta_{k\lambda}(1-m_\lambda^2)\,\delta_{\ell j}(1-m_j^2) + \delta_{kj}(1-m_j^2)\,\delta_{\ell\lambda}(1-m_\lambda^2)$$

$$= \sum_{\langle k\ell\rangle} m_k m_\ell m_\lambda m_j +$$

$$+ \Big(\sum_{\langle k\ell\rangle} m_k m_\ell\Big)\delta_{\lambda j} C_j + \sum_\delta m_{j+\delta}\, m_\lambda C_j + \sum_\delta m_{\lambda+\delta}\, m_j C_\lambda$$

$$+ \sum_\delta C_\lambda m_{\lambda+\delta}\, m_j + \sum_\delta C_j m_{j+\delta}\, m_\lambda +$$

$$+ \sum_\delta C_\lambda\, \delta_{\lambda+\delta,j}\, C_j + C_j\, \delta_{j+\delta,\lambda}\, C_\lambda$$

Se adesso vogliamo sottrarre il calcolo di

$\langle\sigma_i\rangle^{(1)}\langle\sigma_j\rangle^{(1)}$

$$\langle\sigma_i\rangle^{(1)} = \langle\sigma_i\rangle_{eff} - \beta\left[\langle H\sigma_i\rangle^c_{eff} - \langle H_{eff}\sigma_i\rangle^c_{eff}\right] =$$

$$= m_i - \beta\Big[-\frac{J}{2}\sum_{\langle kl\rangle}\left[\langle\sigma_k\sigma_l\sigma_i\rangle_{eff} - \langle\sigma_k\sigma_l\rangle\langle\sigma_i\rangle\right] -$$

$$- \sum_k h_k^{loc}\left[\langle\sigma_k\sigma_i\rangle - \langle\sigma_k\rangle\langle\sigma_i\rangle\right]\Big] \quad \text{per } h=0$$

$$\langle\sigma_k\sigma_l\sigma_i\rangle = \langle(u_k+m_k)(u_l+m_l)(u_i+m_i)\rangle =$$

$$= \langle u_k u_l u_i\rangle +$$

$$+ \cancel{\langle u_k u_l\rangle m_i} + \langle u_k u_i\rangle m_l + \langle u_l u_i\rangle m_k$$

$$+ \cancel{m_i m_j \langle u_k\rangle} + \cdots + m_i m_l m_k$$

$$= \langle u_k u_l u_i\rangle + C_i\delta_{ki} m_l + C_i\delta_{li} m_k + m_i m_l m_k =$$

$$=$$