Exercise #1

due date: 21th October 2019

a) Ergodic flow map: Consider the exercise 6.4 of the Reichl's book:

> ■ **EXERCISE 6.4.** Consider a dynamical flow on the two-dimensional unit square, $0 \leq p \leq 1$ and $0 \leq q \leq 1$, given by the equations of motion, $(dp/dt) = \alpha$ and $(dq/dt) = 1$. Assume that the system has periodic boundary conditions. (a) Show that this flow is ergodic. (b) If the initial probability density at time, $t = 0$, is $\rho(p, q, 0)$, compute the probability density at time, t.

- Find a conserved quantity.
- Find the stationary distribution function.
- Are volumes of phase space conserved?
- Choose an observable f(p,q)
- Is the system ergodic w.r.t. that observable?
- By choosing an ensemble of initial states is the system mixing?

b) Consider the Kac-ring model.

- write a code to calculate the number of black and white balls
- compare the output of the code with the "molecular-chaos" solution given in the lecture and discuss the results.

c) Consider harmonic oscillators of frequency ω in two distinct cases

i. An ensemble of thermalized oscillators described by the density matrix $\hat{\rho} = \sum_n P_n |n> \ <n|$ where $|n>$ are eigenstates of harmonic oscillators and $P_n \propto \exp\left(-\beta E_n\right)$

ii. A pure state $|\psi> \ = \sum_n \sqrt{P_n}|n>$ with the same P_n as in i)

In both case calculate $<x^2>$ as a function of temperature - assuming a Boltzmann distribution for P_n - and comment the results.

d) Consider N classical **independent** one-dimensional harmonic oscillators (mass m_i, frequency ω_i $i = 1, N$) whose initial data are taken according Boltzmann distribution for position and momenta at temperature T. Let them evolve and calculate the following correlation functions:

$C_{i,j}\left(t,t'\right) = \ <p_i\left(t\right)p_j\left(t'\right)>$, $G_{i,j}\left(t,t'\right) = \ <x_i\left(t\right)x_j\left(t'\right)>$ where averages and done on the Boltzmann distribution of initial data.

By assuming a distribution of the oscillator's frequencies $(\omega_i > 0)$ $P\left(\omega\right) \propto \omega^2 \quad \omega < \Lambda$ as well as equal masses for all oscillators calculate in the large N limit:

$C\left(t,t'\right) = \left(1/N\right)\sum_{i,j} C_{i,j}\left(t,t'\right)$, $G\left(t,t'\right) = \left(1/N\right)\sum_{i,j} G_{i,j}\left(t,t'\right)$

find the time evolution of these functions, temperature dependence and dependence on the cutoff Λ .

hint : express the solution for x and p as a function of initial data x(0) and p(0).
