Exercise #2

due date: 4th November 2019

a) deleted

b) Consider the BBGKY evolution equation for the reduced ensemble density of a classical system in the case of non intercating particles but in the presence of an external potential $\Phi\left(\vec{q}\right)$.

Make the hypotesis that velocities are thermalised such that the one particle distribution can be written as

$$\rho^{(1)}\left(\vec{p},\vec{q},t\right) = f\left(\vec{p}\right)u^{(1)}\left(\vec{q},t\right)$$

Demonstrate that the equilibrium distribution for the positional part is

$$u^{(1)}\left(\vec{q}\right) \propto \exp\left[-\beta\,\Phi\left(\vec{q}\right)\right]$$

Now remove the external potential and add a pairwise interaction between particles and assume that again velocities are thermalised such that the two particle distribution can be written as

$$\rho^{(2)}\left(\vec{p}_1,\vec{p}_2\vec{q}_1,\vec{q}_2,t\right) = f\left(\vec{p}_1\right)f\left(\vec{p}_2\right)u^{(2)}\left(\vec{q}_1,\vec{q}_2,t\right)$$

Derive the equation of motion for $u^{(2)}\left(\vec{q}_1,\vec{q}_2,t\right)$

c) Consider a system of classical non-interacting particles enclosed in a 2d circle of radius R. They are at equilibrium at temperature T. They are subject to a potential

$$V\left(r\right) = \frac{1}{2}m\omega_0 r^2 \quad r < r_0$$

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$$V\left(r\right) = \frac{1}{2}m\omega_0 {r^2}_0 = const \quad r > r_0$$

and $r_o < R$

- Determine **and plot** the density of the particle as a function of r/ℓ where ℓ is a suitably defined temperature dependent unit length.
- Determine **and plot** the fraction of particle which are inside the smallest circle as a function of temperature.
- For the two previous quantities discuss the limit $\ell^2 << r_0$ and $\ell^2 >> r_0$