Exercise #4

due date: December 10th

a) Consider a classical perfect gas and calculate the entropy in the canonical ensemble. Compare the result with that given, in the microcanonical ensemble, by the Sakur-Tetrode formula and show the ensemble equivalence in the thermodynamic limit.

b) Consider the following Hamiltonian for N independent spin $\sigma = \pm 1$

$H = -gB\sum_i \sigma_i$

Where B is the external magnetic field along z, g a coupling constant and σ_i the Pauli matrix σ_z at a given site i. Spin operators at different sites commute.

Perform the calculation in the microcanonical ensemble at fixed total **energy**, calculate the entropy. Plot the dimensionless entropy per spin $(S/k_B N)$ as a function of a suitably defined dimensionless energy.

Perform the calculation of the entropy in the canonical ensemble at fixed total **temperature**.

Compare the results of the previous two points by expressing the canonical entropy as a function of the internal energy.

Comment the result.

c) In the grandcanonical ensemble calculate the dimensionless ratio $\beta P/\rho$ where $\rho = N/V$ is the number density by performing a fugacity expansion in $z = \exp(\beta\mu)$ up to second order in z.

Perform the calculation in the case of an interacting classical gas with pair interactions.

Perform the calculation in the case of a quantum perfect gas in the case of Fermi-Dirac and Bose-Einstein statistics.

Express the result in term of the number density in both cases. Comment the two results.

d) Consider a perfect gas of bosonic non-relativistic particles in d dimensions. Discuss the existence of Bose-Einstein condensation as a function of the system dimensionality.