Exercise #1

due date: November 2nd 2020

a) Read the notes about the Kac ring model:

- which set of variables describes a *microscopic* state?
- which set of variables describes the *macroscopic* state?
- write a code to calculate the number of black and white point (you can also do it considering a small size say N=4 and doing the evolution by hand…)
- compare the output of the code with the "molecular-chaos" solution given in the notes and discuss the results.

b1) Consider only one one-dimensional classical harmonic oscillator

- write the Hamilton equation and *plot* a typical trajectory in the phase space
- is the system chaotic?

b2) now consider an ensemble made of replicas of the previous case i.e. one-dimensional classical harmonic oscillator with same ω and mass

- write the Liouville's evolution for and ensemble of such systems using momenta and position as variables
- write the Liouville's evolution for and ensemble of such systems using action-angle variables (amplitude and phase)
- consider an *isoenergetic* ensemble of oscillators (all osc. have energy=E). This ensemble starts with random phases between ϕ_0 and $\phi\_0+\Delta$. In such conditions evaluate $<x(t)>$. Does it tends to a constant value? Is the system ergodic? Is the system mixing?

c) Consider N classical **independent** one-dimensional harmonic oscillators (mass m_i, frequency ω_i $i=1,N$) whose initial data are taken according Boltzmann distribution for position and momenta at temperature T. Let them evolve and calculate the following correlation functions:

$C\_{i,j}(t,t')=< p\_i(t)p\_j(t') >$, $G\_{i,j}(t,t')=< x\_i(t)x\_j(t') >$ where averages and taken on the Boltzmann distribution of initial data.

By assuming a distribution of the oscillator's frequencies ($\omega\_i>0$) $P(\omega)\propto \omega^2 \;\;\; \omega<\Lambda$ as well as equal masses for all oscillators calculate in the large N limit:

$C(t,t')=(1/N)\sum_{i,j} C_{i,j}(t,t')$, $G(t,t')=(1/N)\sum_{i,j} G_{i,j}(t,t')$

find the time evolution of these functions, temperature dependence and dependence on the cutoff Λ.

hint : express the solution for x and p as a function of initial data $x(0)$ and $p(0)$.

d) Consider quantum harmonic oscillators of frequency $\omega}$ in two distinct cases

1. An ensemble of thermalized oscillators described by the density matrix $\hat{\rho}=\sum_n P_n |n><n|$ where $|n>$ are eigenstates of harmonic oscillators and $P_n\propto \exp(-\beta E_n)$
2. A pure state $|\psi>=\sum_n \sqrt{P_n}|n>$ with the same P_n as in i)

In both case calculate $<x^2>$ as a function of temperature - assuming a Boltzmann distribution for P_n - and comment the results.

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