# Exercise #2

due date: 11th November 2020

a) Consider the BBGKY evolution equation for the reduced ensemble density of a classical system in the case of non interacting particles but in the presence of an external potential $\Phi\left(\vec{q}\right)$.

Make the hypotesis that velocities are thermalised such that the one particle distribution can be written as

$$\rho^{(1)}\left(\vec{p}, \vec{q}, t\right) = f\left(\vec{p}\right) u^{(1)}\left(\vec{q}, t\right)$$

Demonstrate that the equilibrium distribution for the positional part is

$$u^{(1)}\left(\vec{q}\right) \propto \exp\left[-\beta\, \Phi\left(\vec{q}\right)\right]$$

Now remove the external potential and add a pairwise interaction between particles and assume that again velocities are thermalised such that the two particle distribution can be written as

$$\rho^{(2)}\left(\vec{p}_1, \vec{p}_2 \vec{q}_1, \vec{q}_2, t\right) = f\left(\vec{p}_1\right) f\left(\vec{p}_2\right) u^{(2)}\left(\vec{q}_1, \vec{q}_2, t\right)$$

Derive the equation of motion for $u^{(2)}\left(\vec{q}_1, \vec{q}_2, t\right)$

b) Read the chapter 3 of the textbook (Kerson Huang, "Statistical Mechanics", Second Edition, Wiley). Try to answer to one of the problems of the chapter at your convenience. If you dare you can try exercise 3.5 and for that it would be useful to read this paper

c) Consider one classical particle in (linear) interaction with a thermalised ensemble of harmonic oscillators as was done in the lectures. Following the notes derive the expression of the *noise term*. Prove that the noise average is

$$< \xi(t) > \ = 0$$

and its correlation function is

$$< \xi\left(t\right) \xi\left(t'\right) > \ = \frac{M\gamma\, \Gamma}{\beta} exp\left(-\ \Gamma\, |t - t'|\right)$$

where the average is taken over a set of initially thermalised oscillators distribution and the associated spectral density is Ohmic.

d) Demonstrate that in the equilibrium Canonical ensemble the entropy can be expressed by

$$S = -k_B tr \rho \log \rho$$

where $\rho$ is the equilibrium density matrix $\exp\left(-\beta H\right)/Z$ and the trace in classical systems should be understood as integrals over $N$ classical d.o.f. divided by $h^{3N} N!$.