Exercise #3

due date: November 30th 2020

a) Consider the stochastic equation for the moment of a particle under the action of external random forces $\xi(t)$ (in one dimension):

$$\dot{p}(t) = -\gamma p(t) + \xi(t)$$

where

$$\langle \xi(t) \rangle = 0$$

$$\langle \xi(t)\xi(t') \rangle = 2M\gamma k_b T\delta(t-t')$$

- Derive without approximation the average mean square displacement

$$\Delta(t) = \langle |x(t) - x(0)|^2 \rangle$$

- Derive the behaviour of $\Delta(t)$ for large and small times and define the time scale above which the behaviour of $\Delta(t)$ is **linear** in time.

b) A polymer can be constructed as a three dimensional random walk where the position of the n+1-th monomer is given by

$$\vec{r}_{n+1} = \vec{r}_n + a\hat{u}_{n+1}$$

where a is the monomer spacing and $\hat{u}$ is a random unit vector. The length of the polymer made by $N+1$ monomers can be estimated by

$$\ell = \sqrt{\langle |\vec{r}_N - \vec{r}_0|^2 \rangle}$$

where the average is taken over the random orientation of the unit vectors $\hat{u}_n$.

- Calculate the ratio ℓ / Na and comment the result.

c) Consider the Fokker-Planck equation in one dimension

$$\frac{\partial}{\partial t}P(x,t) = \frac{\partial}{\partial x}F(x)P(x,t) + \frac{\varepsilon}{2}\frac{\partial^2}{\partial x^2}P(x,t)$$

following the lines of the notes

show that for a potential problem

$$F(x) = -\frac{\partial}{\partial x}V(x)$$

the stationary distribution is given by

$$P(x) = \exp(-2V(x)/\varepsilon).$$

Derive the Maxwell Boltzmann distribution as stationary distribution of the momenta and of the position. For the position distribution use the overdamped approximation in the corresponding Langevin equation.

d) In the microcanonical ensemble consider the following Hamiltonian

$$H = -h\sum_i \sigma_i$$

where $\sigma_i = \pm 1$ and $i = 1, N$.

- Calculate the entropy $S(E, N, h)$ and plot it as a function of E/Nh.
- Calculate the temperature defined as $1/T = \partial S/\partial E$ and explain why it can be **negative**.

Last updated 2020-11-14 17:32:10 CET