

Exercise #4

due date: December 14th 2020

a) Consider a classical perfect gas and calculate the entropy in the canonical ensemble. Compare the result with that given, in the microcanonical ensemble, by the Sakur-Tetrode formula and show the ensemble equivalence in the thermodynamic limit.

b) Consider the following Hamiltonian for N independent spin $\sigma = \pm 1$

$$H = -gB \sum_i \sigma_i$$

Where B is the external magnetic field along z , g a coupling constant and σ_i the Pauli matrix σ_z at a given site i . Spin operators at different sites commute.

Perform the calculation in the microcanonical ensemble at fixed total **energy**, calculate the entropy. Plot the dimensionless entropy per spin ($S/k_B N$) as a function of a suitably defined dimensionless energy.

Perform the calculation of the entropy in the canonical ensemble at fixed total **temperature**.

Compare the results of the previous two points by expressing the canonical entropy as a function of the internal energy.

Comment the result.

c) In the grandcanonical ensemble calculate the dimensionless ratio $\beta P/\rho$ where $\rho = N/V$ is the number density by performing a fugacity expansion in $z = \exp(\beta\mu)$ up to second order in z .

Perform the calculation in the case of an interacting classical gas with pair interactions.

Perform the calculation in the case of a quantum perfect gas in the case of Fermi-Dirac and Bose-Einstein statistics.

Express the result in term of the number density in both cases. Comment the two results.

d) Consider two one particle states made by one-dimensional gaussians

$$g_1(x) = A \left(\exp \left(-\frac{(x - x_0)^2}{2\sigma^2} \right) \right)^{1/2}$$

$$g_2(x) = A \left(\exp \left(-\frac{(x + x_0)^2}{2\sigma^2} \right) \right)^{1/2}$$

with A being a normalisation constant. Using these states write a possible state for 2 fermions and 2 bosons in state 1 or 2 neglecting the spin component. Calculate the average distance $\langle x_1 - x_2 \rangle$ as a function of x_0 (you can choose $\sigma^2 = 1$). Comment the results.