Exercise #5

due date: 11 Jan 2021

a) Consider the perfect Fermi and Bose gas with a general single particle dispersion $\varepsilon(p) = a|p|^b$. Determine:

- the density of the states (DOS)

$$N(\epsilon) = \frac{V}{h^3} \int d^3p\delta(\epsilon - \epsilon(p))$$

- the relation between PV and U as a function of the DOS parameters

b) Consider a perfect gas of bosonic non-relativistic particles in d dimensions. Discuss the existence of Bose-Einstein condensation as a function of the system dimensionality. Using the relation $PV = 2U/3$ prove that when $T < T_{BE}$ the pressure depends only on temperature and find its temperature dependence.

c) Prove that the equilibrium single-particle density matrix

$$\hat{\rho} = \frac{\exp(-\beta H^{(1)})}{Z^{(1)}}$$

for a free particle is

$$\langle x|\exp(-\beta H)|y\rangle / Z^{(1)} = A\exp(-(x-y)^2/a\lambda^2)$$

where $Z^{(1)}$ is the single particle partition function, λ is the De Broglie wavelength and $|x>, |y>$ position eigenstates. Find the normalization A for a tree dimensional space and the numerical coefficient a. Comment the results. Compare the result for $\langle x|\rho|y\rangle$ to that for $\langle p|\rho|p'\rangle$ with p and p' are three dimensional wave vectors.

d) A team of 12 reindeers pull the sleigh of QuantaClaus. Write a Hamiltonian for the system in second quantisation and consider spinless reindeers.

- Does the statistics of Quanta matter?
- Could reindeers be $s = 1/2$ fermions?