

Exercise #2

due date: November 7 2022

a) Consider N classical **independent** one-dimensional harmonic oscillators (mass m_i , frequency ω_i $i=1, N$) whose initial data are taken according Boltzmann distribution for position and momenta at temperature T . Let them evolve and calculate the following correlation functions:

$C_{\{i,j\}}(t,t') = \langle p_i(t)p_j(t') \rangle$, $G_{\{i,j\}}(t,t') = \langle x_i(t)x_j(t') \rangle$ where averages are taken on the Boltzmann distribution of initial data.

By assuming a distribution of the oscillator's frequencies ($\omega_i > 0$) $P(\omega) \propto \omega^2$; $\omega < \Lambda$ as well as equal masses for all oscillators calculate in the large N limit:

$C(t,t') = (1/N) \sum_{\{i,j\}} C_{\{i,j\}}(t,t')$, $G(t,t') = (1/N) \sum_{\{i,j\}} G_{\{i,j\}}(t,t')$

find the time evolution of these functions, temperature dependence and dependence on the cutoff Λ .

hint(1) : express the solution for x and p as a function of initial data $x(0)$ and $p(0)$.

hint(2) : express the solution for x and p as a function of amplitude and phase and then express the Boltzmann distribution in the same variables.

b1) Read the notes. Show the the interaction-dependent term $\hat{L}$ defined in page 8 is hermitian.

b2) **optional** Consider the generalised Gauss theorem and show that if $\rho(x) = \Delta_\Omega(x)$ is the characteristic function of the set Ω the standard Gauss theorem follows.

c) Consider the BBGKY evolution equation for the reduced ensemble density of a classical system in the case of non interacting particles but in the presence of an external potential $\Phi(\vec{q})$.

Make the hypothesis that velocities are thermalised such that the one particle distribution can be written as

$$\rho^{(1)}(\vec{p}, \vec{q}, t) = f(\vec{p}) u^{(1)}(\vec{q}, t)$$

Demonstrate that the equilibrium distribution for the positional part is

$$u^{(1)}(\vec{q}) \propto \exp[-\beta \Phi(\vec{q})]$$

d) Read the chapter 3 of the textbook (Kerson Huang, "Statistical Mechanics", Second Edition, Wiley). Try to answer to one of the problems of the chapter at your convenience. If you dare you can try exercise 3.5 and for that it would be useful to read this paper

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