Exercise #3

due date: December 4th 2022

a) Following the lines of the notes derive the statistical properties of the noise term $\xi(t)$ as a result of the equilibrium distribution for the oscillators. Perform explicitly the $N\rightarrow \infty$ limit using the Ohmic spectral density. Does the noise correlation function behaviour depends on the $\omega\rightarrow 0$ behviour of the spectral density? You can **optionally** answer to this question trying to guess a spectral density which goes as ω^α for small ω.

b) Consider the stochastic equation for the moment of a particle under the action of external random forces $\xi(t)$ (in one dimension):

$\dot{p}(t)=-\gamma p(t) + \xi(t)$

where

$\langle \xi(t) \rangle = 0$

$\langle \xi(t) \xi(t')\rangle = 2 M \gamma k_b T \delta(t-t')$

- Derive without approximation the average mean square displacement

$\Delta(t)=\langle |x(t)-x(0)|^2 \rangle$

- Derive the behaviour of $\Delta(t)$ for large and small times and define the time scale above which the behaviour of $\Delta(t)$ is **linear** in time.

c) Consider the Fokker-Planck equation in one dimension

$\frac{\partial}{\partial t} P(x,t) = \frac{\partial}{\partial x} F(x)P(x,t) +\frac {\epsilon}{2}\frac{\partial^2}{\partial x^2} P(x,t)$

following the lines of the notes

show that for a potential problem

$F(x) = -\frac{\partial}{\partial x} V(x)$

the stationary distribution is given by

$P(x)=\exp(-2V(x)/\epsilon)$.

Consider the multidimensional Fokker-Plank equation and derive the equation for position and momenta in the case of a particle moving in one dimension under the action of a potential force. Comment the equation and compare it with the Boltzmann equation.

Derive the Maxwell Boltzmann distribution as stationary joint distribution momenta and of the position.

d) A polymer can be constructed as a three dimensional random walk where the position of the n+1-th monomer is given by

$\vec{r}_{n+1}=\vec{r}_{n}+a\hat{u}_{n+1}$

where a is the monomer spacing and $\hat{u}$ is a random unit vector. The length of the polymer made by $N+1$ monomers can be estimated by

$\ell= \sqrt{\langle |\vec{r}_{N}-\vec{r}_{0}|^2 \rangle}$

where the average is taken over the random orientation of the unit vectors $\hat{u}_n$.

- Calculate the ratio ℓ/Na and comment the result.

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