Exercise #3

due date: November 20th 2023

a) Consider two one particle states made by one-dimensional gaussians

$g_1(x)=A\left (\exp(-\frac{(x-x_0)^2}{2\sigma^2}) \right)^{1/2}$

$g_2(x)=A\left (\exp(-\frac{(x+x_0)^2}{2\sigma^2}) \right)^{1/2}$

with A being a normalisation constant. Using these states write a possible state for 2 fermions and 2 bosons in state 1 or 2 neglecting the spin component. Calculate the average distance $\langle | x_1-x_2 | ^2 \rangle$ as a function of x_0 (you can choose $\sigma^2=1$). Comment the results.

b) Consider the perfect Fermi and Bose gas with a general single particle dispersion $\epsilon(p)= a |p|^b$. Determine:

- the density of the states (DOS)

$$N(\epsilon) = \frac{V}{h^3} \int d^3 p \delta(\epsilon - \epsilon(p))$$

- the relation between PV and U as a function of the DOS parameters

c) Consider a perfect gas of bosonic ultra-relativistic particles in 3 dimensions with vanishing chemical potential (black body). Calculate internal energy as a function of temperature and using the relation derived in the previous point prove that the radiation pressure is independent of volume and is proportional to T^4.

d1) Prove that the equilibrium single-particle density matrix for Boltzmann particle is

$$\hat{\rho} = \frac{\exp(-\beta H^{(1)})}{Z^{(1)}}$$

for a free particle is

$$\langle x| \exp(-\beta H) |y\rangle / Z^{(1)} = A \exp(-(x-y)^2/a\lambda^2)$$

where $Z^{(1)}$ is the single particle partition function, λ is the De Broglie wavelength and $|x>,|y>$ position eigenstates (in 3 dimensions). Find the normalization A for a tree dimensional space and the numerical coefficient a. Comment the results. Compare the result for $\langle x| \rho |y \rangle$ to that for $\langle p| \rho |p^\prime \rangle$ with p and $p^\prime$ are three dimensional wave vectors.

d2) *Optional* Consider the equilibrium single-particle density matrix in the grand-canonical ensemble for massive non-relativistic non-interacting Bose particles. Write its form in momentum and position (3d) representation. Comment the limit

$\mu\rightarrow 0^{-}$ for the density matrix in the coordinate representation. Using the fugacity expansion to all order or the integral expression of the density matrix write a code to evaluate numerically it when $\mu < 0$.

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