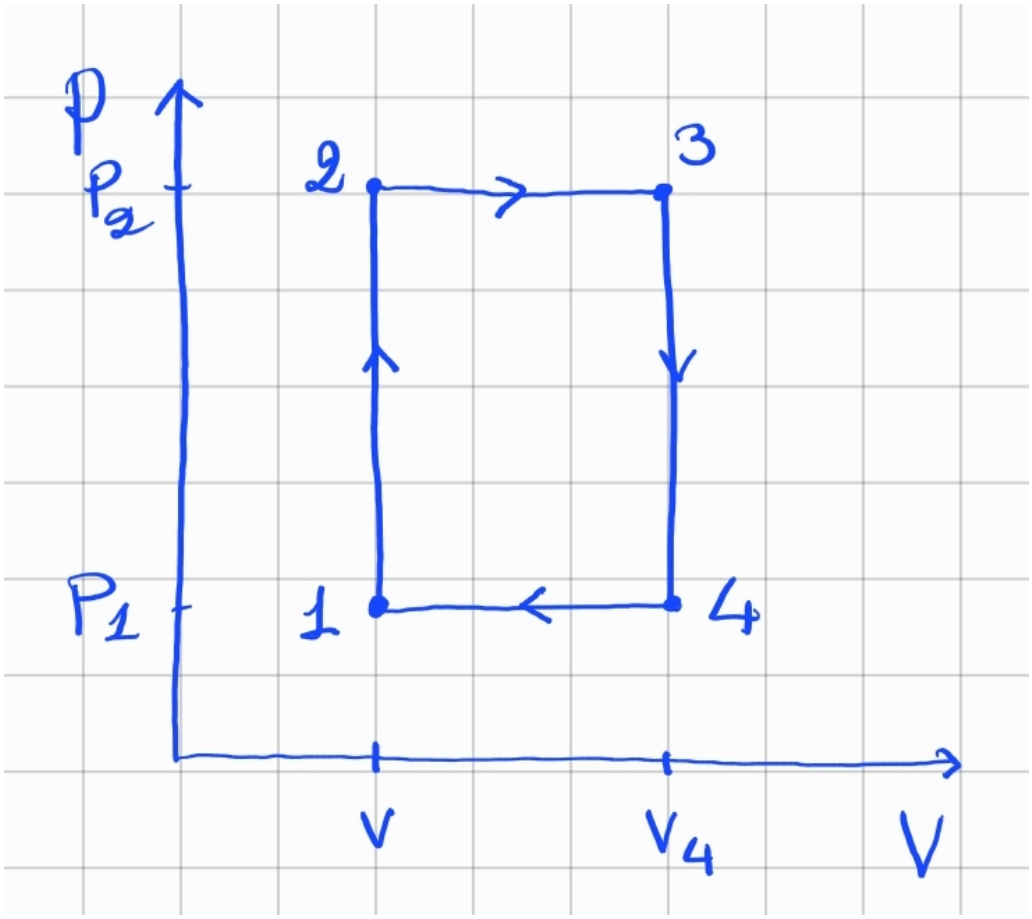
Exercise #1

due date: October 21 2024

a) Consider the following thermodynamic cycle for a perfect gas:



- Draw the cycle in the T-S plane.
- Calculate the total work exerted by the system (W).
- Calculate the total heat exchanged by the system (Q).
- Calculate the efficiency $\eta={W}/{Q_{abs}}$ where Q_{abs} is the absorbed heat.

b) Consider only one one-dimensional classical harmonic oscillator

- write the Hamilton equation and *plot* a typical trajectory in the phase space

now consider an ensemble made of replicas of the previous case i.e. one-dimensional classical harmonic oscillator with same ω and mass

- write the Liouville's evolution for and ensemble of such systems using momenta and position as variables
- consider an *isoenergetic* ensemble of oscillators (all osc. have energy=E). This ensemble starts with random phases between ϕ_0 and $\phi_0+\Delta$. In such conditions evaluate $<x(t)>$. Does it tends to a constant value?

c) Consider a classical perfect gas and calculate the entropy in the canonical ensemble. Compare the result with that given, in the microcanonical ensemble, by the Sackur-Tetrode formula and show the ensemble equivalence in the thermodynamic limit.

d) Consider the following Hamiltonian for N independent spin $\sigma=\pm 1$

$H=-gB\sum_i \sigma_i$

Where B is the external magnetic field along z, g a coupling constant and σ_i the Pauli matrix σ_z at a given site i. Spin operators at different sites commute.

Perform the calculation in the microcanonical ensemble at fixed total **energy**, calculate the entropy. Plot the dimensionless entropy per spin ($S/k_B N$) as a function of a suitably defined dimensionless energy.

Perform the calculation of the entropy in the canonical ensemble at fixed total **temperature**.

Compare the results of the previous two points by expressing the canonical entropy as a function of the internal energy.

Comment the result.

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