Numerical exercise #1

tarball Oscillators_v3.1.tgz

parameter file Oscillator.in

The code integrates the equation of motion of the whole system using leap frog Verlet algorithm.

N oscillators $m_i = 1$

1 particle $M = 1$

frequencies $\omega_i = i\Delta$ $\quad \omega_{MAX} = N\Delta$ by input

How to choose the couplings c_i from Ohmic distribution

$$J(\omega) = \frac{\pi}{2}\sum_1^N{}_i \frac{c_i^2}{m_i\omega_i}\delta(\omega-\omega_i) = M\gamma \frac{\omega\Gamma^2}{\omega^2+\Gamma^2}$$

(1) $\qquad$ (2)

Integrate (1) & (2) around a given frequency ω_i

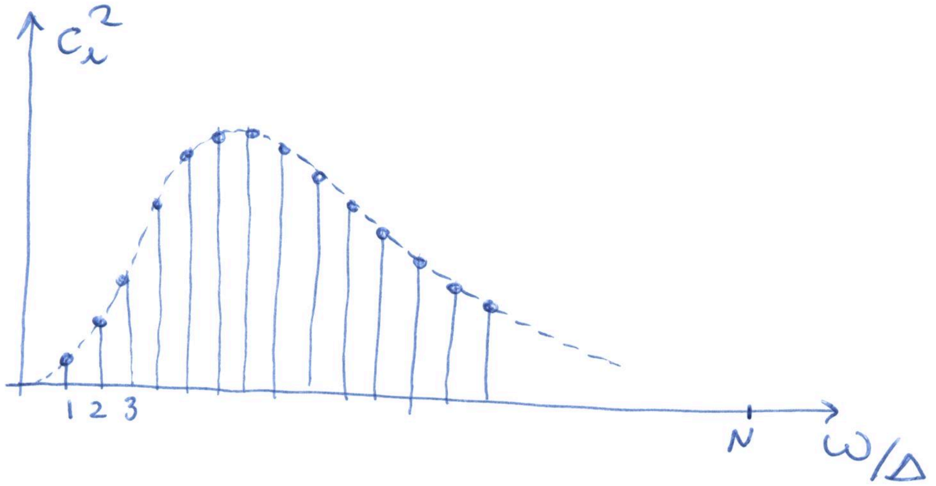
1) $$\frac{\pi}{2}\sum_1^N{}_k \frac{c_k^2}{m_k\omega_k}\int_{\omega_i-\Delta/2}^{\omega_i+\Delta/2} d\omega\,\delta(\omega-\omega_k) = \frac{\pi}{2}\frac{c_i^2}{m_i\omega_i}$$

2) $$M\gamma\int_{\omega_i-\Delta/2}^{\omega_i+\Delta/2} d\omega \frac{\omega\Gamma}{\omega^2+\Gamma^2} \simeq M\gamma\frac{\omega_i\Gamma}{\omega_i^2+\Gamma^2}\Delta$$

choosing $m_i = 1$

$$c_i^2 = 2\gamma\Delta\frac{\omega_i^2\Gamma^2}{\pi(\omega_i^2+\Gamma^2)} \qquad c_i \sim \sqrt{\Delta} = \sqrt{\frac{\omega_{MAX}}{N}}$$

each coupling is small the collective coupling is $\mathcal{O}(1)$



Numerical exercises: run the program varying time steps and bath temperature

Bath temperature is defined by the variance of the gaussian distribution of $y_l(0)$ & $p_l(0)$

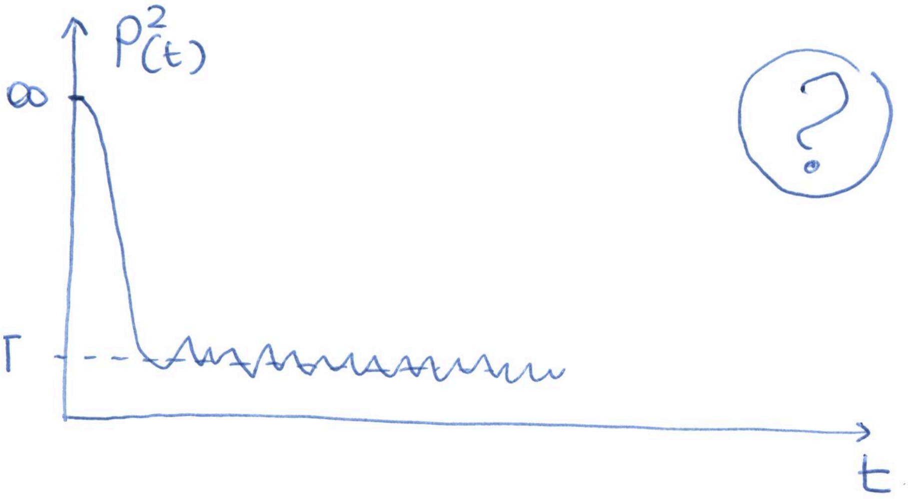
$$P(y_i(0)) \propto e^{-\beta \frac{1}{2}\omega_l^2 y_l^2} \qquad P(p_i(0)) \propto e^{-\beta \frac{p_i^2}{2}}$$

$$\beta = \frac{1}{T} \quad (k_B = 1)$$

If system thermalizes then

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T dt'\, P^2(t') = \langle P^2 \rangle = T$$

- independent on initial condition
- independent on $V(x)$



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