# Problems: set 3

1. Consider a collection of $N$ free massless left spinors and $N$ free massless right spinors. Write the action and show that it is invariant under the global symmetry $U_L(N) \times U_R(N) = SU_L(N) \times SU_R(N) \times U_L(1) \times U_R(1)$ and determine the corresponding Noether currents. Take for instance the $SU_L(N)$ current $J^\mu_{L\,a}$ and argue that it can be written in terms of vector and axial parts according with

$$J^\mu_{L\,a} = J^\mu_{V\,a} + J^\mu_{A\,a}\,, \qquad J^\mu_{V\,a} = \frac{i}{2}\,\bar\psi\,\gamma^\mu\,T_a\,\psi \qquad J^\mu_{A\,a} = -\frac{i}{2}\,\bar\psi\,\gamma^\mu\,\gamma^5\,T_a\,\psi\,. \tag{1}$$

Do the same for all the right handed and left handed currents. Find the field transformations corresponding to the axial and vector currents.

2. Referring to the previous problem take $N = 2$ and add the following mass term

$$\mathcal{L}_M = -\bar\psi\,M\,\psi\,, \qquad M = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}\,. \tag{2}$$

Compute the divergencies of the currents found in the previous problem, namely the currents relative to $SU_L(2) \times SU_R(2) \times U_L(1) \times U_R(1)$ and the corresponding axial and vector parts. Interpret the result.

3. Consider the following pattern for the symmetry breaking $U(1) \to Z_2$ realized by using a complex scalar field which gets a vacuum expectation value. Write the conserved current $J^\mu$ by using the Kibble parametrization of $\phi$ in terms of $\rho$ and the Goldstone mode $\xi$. Compute at the leading order in perturbation theory the following matrix element

$$< 0|J^\mu(x)|p >\,, \tag{3}$$

where $|0>$ is the vacuum and $|p>$ is a state corresponding to a Goldstone boson with 4-momentum $p^\mu$. Argue that such a matrix element can be different form zero only in the broken phase. Thus, in the broken phase, the current connects the vacuum with a Goldstone boson state. Find the consequence of the current conservation.