Esercises: set 1

1. Take a complex linear space of dimension 3 and a orthonormal basis $\{|n>, n = 1, \cdots, 3\}$ An operator A acts as

$$A|m>= i\sum_{n=1}^{3} \epsilon_{1mn}\,|n> \ .$$

   The Levi-Civita ϵ symbol is such that $\epsilon_{ijk} = -\epsilon_{jik} = -\epsilon_{ikj} = -\epsilon_{kji}$ with $\epsilon_{123} = 1$. Determine the representation of A with respect the given basis and whether A is hermitian.

2. Compute the commutator of $[p,\ f(q)]$ where p is the momentum operator, q is the position operator and f is differentiable function.

3. Given two operators A and B, by using complete orthonormal basis find the explicit form of $\mathrm{Tr}(A)$ and show that $\mathrm{Tr}(A\,B) = \mathrm{Tr}(B\,A)$.

4. Given and operator $\hat{A}$ with a complete orthonormal basis $\{|n>\}$ of eigenvectors, $\hat{A}|n>= z_n\,|n>$; show that the projection operator $P_n = |n>< n|$ in the "direction" $|n>$, is such that $P_n^2 = P_n$ and $A - \sum_n z_n |n>< n| = 0$.

5. Given an unitary operator U, vector $|\psi>$ and an operator$\hat{A}$, consider the expectation value $\bar{A} =< \psi|A|\psi>$ of A on $|\psi>$. Defining $|\psi'>= U|\psi>$ and $A_U = UAU^{-1}$, show that $\bar{A} =< \psi'|A_U|\psi'>$. Finally compute $[A_U, B_U]$ in terms of $C = [A, B]$

6. The operator A has the following representation with respect the orthonormal basis $\{|1>, |2>, |3>\}$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} .$$

   - Is A hermitian ?
   - Determine the eigenvalues λ_i and the eigenvectors $|\lambda_i>$ of A.
   - Given the state vector

$$|\psi>= \frac{1}{\sqrt{2}}\,(|2>+|3>) \ .$$

     Compute the probability $P(\lambda_i)$ of measuring λ_i on the state $|\psi>$.