Exercises: set 3

1. Consider a particle of mass m confined on line and subject to the following potential $V(x) = v_0\delta(x)$. Discuss the existence of bound states by using that V can be understood as the limit of

$$U(x) = \begin{cases} 0 & x < -\frac{L}{2} \text{ and } x > \frac{L}{2} \\ U_0 & x \in [-\frac{L}{2}, \frac{L}{2}] \end{cases};$$

   when $L \to 0$ and $|U_0| \to \infty$ with $v_0 = LU_0$ constant.

2. Consider a particle of mass m confined on line and subject to the following potential $V(x) = v_0 [\delta(x + x_0) + \delta(x - x_0)]$ with x_0 positivo. Discuss the existence of bound states and find the eigenvalues and eigenstates of the Hamiltonian.

3. Consider an harmonic un oscillator armonico with mass m and "frequency" ω. At $t = 0$ is known that an energy measure can give the outcome E_0 or E_1, where E_0 and E_1 are is the energy of the fundamental state and the first excited state respectively.

   - Determine the most general state at the initial time $t = 0$ consistent with the given informations.
   - Determine the average values the operators q and e p at time t on the state $|\psi, t>$ by using both the Schrodinger and Heisenberg pictures. In addition finds the values of parameters entering $|\psi, t = 0>$ such $< q >$ is maximum at time $t = 2\pi/\omega$.
   - Show that the quantum version of the Virial theorem holds, namely: $< T >=< U >= E/2$, where T is the kinetic energy operator and , U is the potential energy l'operatore. The average is taken on a generic eigenstate of the harmonic oscillator.